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Reconstruction Of Digital Holograms Of Large Objects By Convolution Algorithm

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RESUMEN

La reconstrucción numérica de los hologramas registrados digitalmente – llamada holografía digital (HD) – ha sido denominada una re-invención de la holografía. La HD supera el procesamiento químico de la holografía óptica y la provee de la manipulación numérica de los campos ópticos reconstruidos. Estos campos reconstruidos son obtenidos modelando numéricamente la difracción de la onda que llega sobre el holograma registrado; se han encontrado diferentes aproximaciones para evaluar tal proceso de difracción, algunos de ellos son la transformada discreta de Fresnel, el espectro angular y la convolución. La aproximación por convolución es de gran interés dado que permite el cálculo por tres transformadas rápidas de Fourier logrando altas velocidades de reconstrucción. Su principal característica es que el tamaño del pixel de registro es igual al de reconstrucción, lo cual limita su aplicación a la reconstrucción con objetos de pequeñas dimensiones. En este trabajo nosotros presentamos una aproximación simple para extender el rango de aplicación del método de convolución para reconstruir objetos grandes; se presentan resultados experimentales para validar el método propuesto.
Palabras Clave: Convolución; Reconstrucción de objetos grandes; Integral de difracción.

ABSTRACT

Numerical reconstruction of digitally recorded holograms -named digital holography (DH)- has meant a re-invention of holography. DH overcomes the wet-chemical process of holography and provides the latter with numerical manipulation of reconstructed optical fields. These fields are obtained by numerically modelling the diffraction of a reconstructing wave impinging upon the recorded hologram; different approaches are found to evaluate such diffraction process, discrete Fresnel transform, angular spectrum and convolution are some of them. The convolution approach is of great interest since it allows for the full calculation by three Fast-Fourier-Transforms enabling high speed reconstructions. Its main characteristic is that the recording and reconstruction pixels equal in size, which limits its application to reconstruction of objects with small dimensions. In this work we present a simple approach to extend the range of application of the convolution method to reconstruct holograms of large object; experimental results are shown to validate the proposed method.
Keywords: convolution; large-object-reconstruction; diffraction-integral.

1. INTRODUCCIÓN

Numerical reconstruction of digital recorded holograms, named Digital Holography (DH) is supported on the same foundations as of optical holography; it can be modelled into two diffraction steps. Firstly, an object located at a plane $z = 0$ is coherently illuminated; the scattered optical field interferes with a reference field on the so called hologram plane placed at a distance $z = d$. At this plane, a 2D-discrete detector (CCD or CMOS camera) is placed such that a sampled version of the interference pattern $I\left(x_h, y_h\right)$ is recorded and stored in a computer. The recorded intensity carries on information about i) the amplitudes of the reference and scattered, known as zeroth-diffraction order, and ii) the interference term that provides conjugate versions of the scattered wave field, i.e. the object itself[1].

The second step has as objective to recover the real version of the scattered wave field. It is achieved by diffracting $I\left(x_h, y_h\right)$ when it is illuminated by the conjugated reference wave. We assume that this reference wave has wavelength λ, amplitude E_0, spatial distribution $r^*\left(x_h, y_h\right)$ and r_0 wavefront radius; the hologram extends over an area Σ.

The hologram reconstruction can be then performed by calculating the Fresnel-Kirchhoff diffraction formula[2] :

$$E(x_i, y_i, z) = -\frac{i}{2\lambda}\frac{E_0 \exp(ikr_0)}{r_0}\iint_{\Sigma} I(x_h, y_h)\, r^*(x_h, y_h)\,\frac{\exp(iks)}{s}\,(1+Cos\chi)\, dx_h dy_h \qquad (1)$$

where $i=\sqrt{-1}$, $k=\frac{2\pi}{\lambda}$, $(1+Cos\chi)$ is the inclination factor, χ equals the smallest angle between the normal vector to the hologram at the point (x_h, y_h) and the vector from this point to the reconstruction one (x_i, y_i, z) ; $\chi \to 0$ for small numerical aperture applications. The distance s between the each pixel on the hologram and reconstruction planes is given by

$$s = \sqrt{z^2 + (x_h - x_i)^2 + (y_h - y_i)^2}\,. \qquad (2)$$

In many practical applications the reference wave is a homogeneous plane wave in amplitude and phase impinging perpendicular to the hologram plane such that it can be represented as a constant field of amplitude E_0. This simplification and small numerical apertures will be considered on the text to follow.

Different approaches to calculate equation (1) are found elsewhere; Fresnel approach relies on the parabolic approximation of the distance $s \approx z\left[1+\frac{1}{2}\left(\frac{x_h-x_i}{z}\right)^2+\frac{1}{2}\left(\frac{y_h-y_i}{z}\right)^2\right]$ for the phase and $s \approx z$ for the amplitude. These approximations transform equation (1) into:

$$E(x_i, y_i, z) = \frac{E_0 \exp(ikz)}{i\lambda z}\exp\left[\frac{i\pi}{\lambda z}(x_i^2 + y_i^2)\right] \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x_h, y_h)\exp\left[\frac{i\pi}{\lambda z}(x_h^2 + y_h^2)\right]\exp\left[-\frac{i\,2\pi}{\lambda z}(x_h x_i + y_h y_i)\right]dx_h dy_h\,. \qquad (3)$$

Equation (3) can be numerically calculated by the discrete Fourier transform of the product recorded hologram $I(x_h, y_h)$ times the Fresnel phase $\exp\left[\frac{i\pi}{\lambda z}(x_h^2 + y_h^2)\right]$ at the hologram plane. When the hologram has N_x, N_y pixels, the sizes of the reconstructed $\Delta x_i, \Delta y_i$ and recorded $\Delta x_h, \Delta y_h$ pixels are related by $\Delta x_i = \frac{\lambda z}{N_x \Delta x_h}, \Delta y_i = \frac{\lambda z}{N_y \Delta y_h}$ such that the total reconstructed area equals to $x_i y_i = \frac{(\lambda z)^2}{\Delta x_h \Delta y_h}$.

Another different approach to evaluate equation (1), relies on the writing of such an equation as a superposition integral:

$$E(x_i, y_i, z) = \iint h(x_i, y_i; x_h, y_h)\, I(x_h, y_h)\, dx_h dy_h \qquad (4)$$

where $h(x_i, y_i; x_h, y_h)$ is the impulse response of the free space given by:

$$h\left(x_i, y_i; x_h, y_h\right) = \frac{E_0}{i\lambda}\frac{\exp\left(iks\right)}{s}$$
$$= \frac{E_0}{i\lambda}\frac{\exp\left(ik\sqrt{z^2 + \left(x_h - x_i\right)^2 + \left(y_h - y_i\right)^2}\right)}{\sqrt{z^2 + \left(x_h - x_i\right)^2 + \left(y_h - y_i\right)^2}} . \quad (5)$$

The evaluation of equation (4) is carried out by accounting the convolution property of Fourier transforms[3]: i) $h\left(x_i, y_i; x_h, y_h\right)$ and $I\left(x_h, y_h\right)$ are Fourier transformed; ii) These Fourier transforms are pixel-wise multiplied; iii) $E\left(x_i, y_i, z\right)$ equals the inverse Fourier transform of such a product. At difference from the Fresnel approach that produces the scattered field $E\left(x_i, y_i, z\right)$ on the spatial frequency space, the convolution approach produces it on the spatial domain. For this reason the pixel size of the reconstructed scattered wave equals that of the recorded hologram. The total reconstructed area will be the given by $x_i y_i = N_x \Delta x_h N_y \Delta y_h$.

2. RECONSTRUCTION OF DIGITAL HOLOGRAM OF LARGE OBJECTS

The different approaches to reconstruct digital holograms have found their scope of application based upon the careful choice of the reconstruction distance such the no fast changes of phase happen, that would ruin the reconstruction process. Another factor to consider on the choice of the reconstruction algorithm is the size of the reconstructed image field. Thanks to the reasonable reconstructed area of the Fresnel approach this approach has fewer constraints than the convolution approach. The latter suffers of the restrictions that the largest size $x_i y_i = N_x \Delta x_h N_y \Delta y_h$, and pixel size of the reconstructed image $\Delta x_i = \Delta x_h ; \Delta y_i = \Delta y_h$ equal those of the recording device.

Figure 1 shows the reconstruction of one hologram done by Fresnel approach (panel A) and by the convolution one (panel B). Spatial filtering on the Fourier domain[1] has been applied for all the reconstruction presented in this work. With this well known procedure, the inconvenient effects of the twin image and the zeroth-diffraction-order have been removed from the reconstructed images.

The hologram was recorded by a CCD camera with 780 x 780 square pixels of 11 µm side. The illumination wavelength was 633 nm and the object was placed at a distance of 1.05 m from the camera; while the hologram is reconstructed with the Fresnel approach these parameters lead to a reconstructed pixel size $\Delta x_i = \Delta y_i = 77.5$ µm and a reconstructed area of $\Delta x_i \Delta y_i = 60.4$ x 60.4 mm^2 when the reconstruction is done with the convolution approach these values equal to $\Delta x_i = \Delta y_i = 11$ µm and $\Delta x_i \Delta y_i = 8.58$ x 8.58 mm^2. Due to the limited reconstructed area of the convolution approach, it is not possible to have the full view of the die that is seem in panel A. However, the smaller pixel size of the reconstructed image for the convolution approach, could lead to a better resolved image if the reduced field of view is somehow increased.

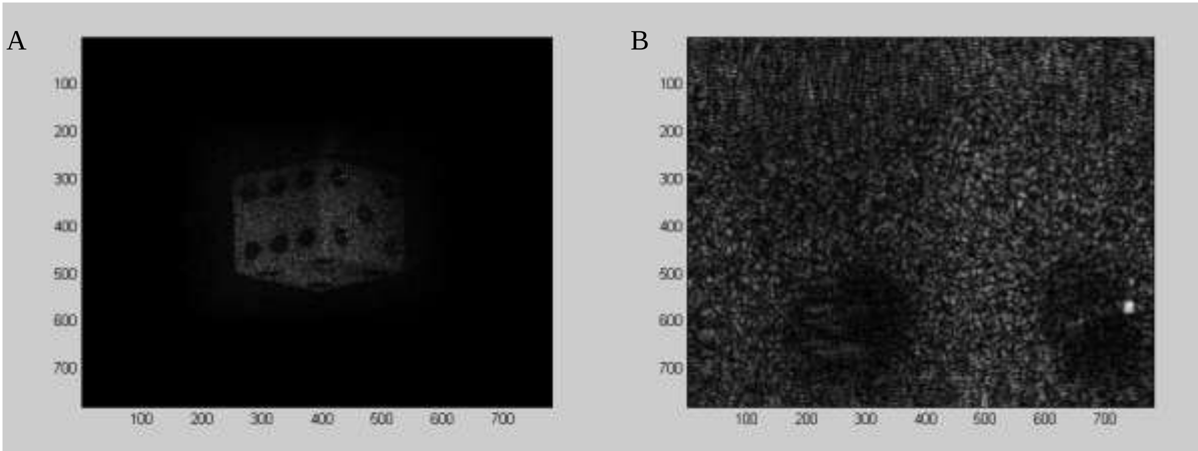

Figure 1. Reconstruction of one hologram with the Fresnel approach, panel A and the convolution method, panel B.

The field of view of the convolution approach can be enlarged by increasing the number of pixels of the input hologram. Since the number of pixels of the CCD camera is set from factory, zero padding of the hologram can help to increase such a field of view[4]. In Figure 2 the original hologram reconstructed on Figure 1, has been padded to 2048 x 2048 pixels and thereafter reconstructed by the two mentioned approaches, for fair comparison.

In both panels of figure 2 the full image of the die is reproduced, but the 4 megapixels of the reconstructed image are used on different ways. In panel A, the Fresnel approach averages the reconstructed image and the die itself is restricted to a small area of the whole image. The new pixel size of the reconstructed image is reduced to $\Delta x_i = \Delta y_i = 29.5$ µm, meaning reduction of the speckle size while keeping the same field of view, which leads to an apparent increase of the resolution. The convolution approach, panel B, shows an enlargement of the field of view that fits the whole image of the die. Since the pixel size is kept fixed, the convolution approach could be understood as a zoom effect of the Fresnel reconstructed image without the need of any digital average or interpolation while keeping speckle size given by the pixel size of the recording device. One could be tempted to claim increase of the lateral resolution with the reconstruction by the convolution approach; however as in any other imaging system, the resolution is given by the size of the aperture (CCD camera in this case), wavelength and observation distance. Since these parameters are the same on both reconstruction methods, the physical lateral resolution is the same and only the zoom effect takes place.

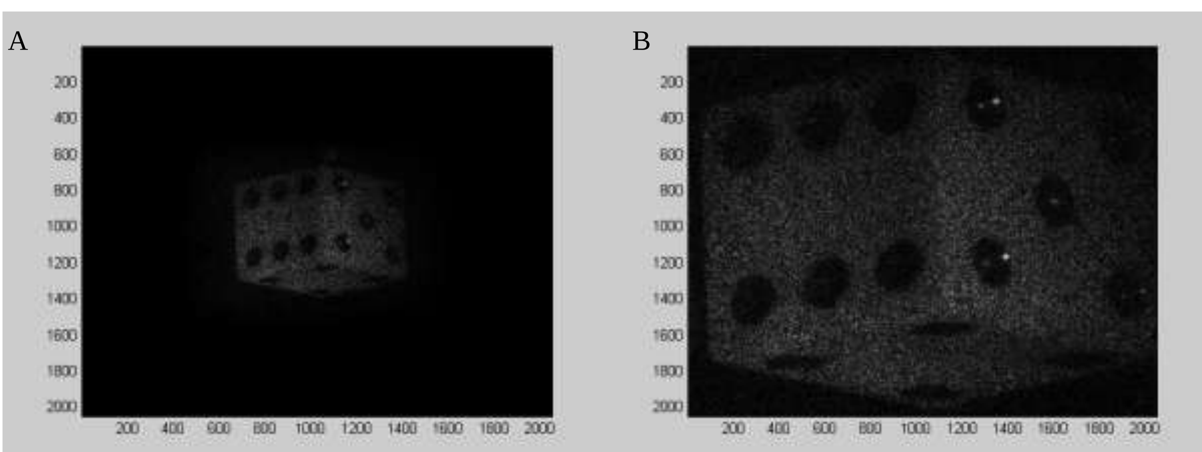

Figure 2. Reconstruction of the same hologram in Figure 1 after padded to 2048 x 2048 pixels; panel A Fresnel approach, and panel B convolution method.

To better illustrate this later idea, Fresnel and convolution approaches have been applied to reconstruct holograms of a USAF 1951 resolution test target. The holograms were recorded at a distance of 40 cm with a CCD camera with 640 x 480 pixels and pixels of 7.4 x 7.2 µm^2; the target 1.5 cm side was illuminated with a wavelength of 633 nm. These parameters mean that in order to reconstruct the whole target with the convolution approach one must pad the original holograms to at least 2048 pixels.

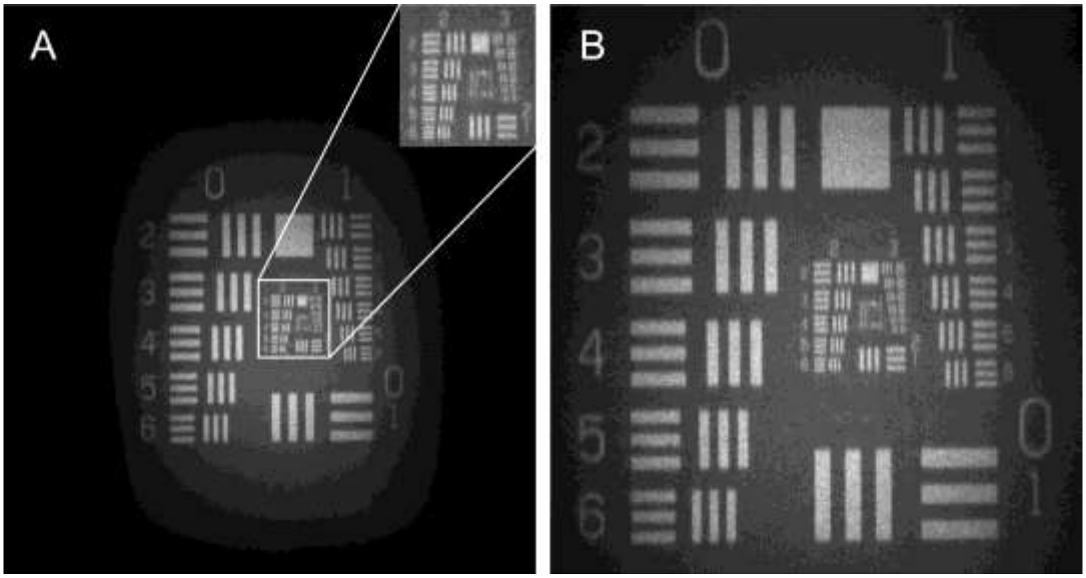

Figure 3. Reconstruction of a USAF 1951 test target; panel A Fresnel approach, and panel B convolution method. The speckle noise has been reduced by using the uncorrelated superposition of holograms[5].

When considered the whole field of view and compared panel A and B of Figure 3, it is apparent that there is an increase of the lateral resolution by reconstructing with the convolution approach, see panel B. However a closer look to the inset in panel A, where a zoom of the region of interest it is shown, one can see the same order of resolution that in panel B. The advantage of doing the reconstruction of the hologram via the convolution approach lies on the fact that this methodology produces reconstruction that resemble the result of digital zooming reconstruction made by other approaches; for this reason the main field of application of this methodology is the reconstruction of holograms of field of particles or in-line holograms of microscopic objects[6].

CONCLUSION

Among the different options to numerically reconstruct the optical field from digitally recorded holograms, Fresnel's and convolution approaches are the most used. The features of each one have been the key point to determine which one to employ for particular experiments. In this paper, the apparent limitation of the convolution approach of being meant to reconstruct only objects with dimensions equal to recording device, it is used to produce a zoom effect of the reconstructed field. To avoid the constraint of the reconstructed size a simple approach of zero padding is used and objects larger than the dimensions of the recording device are successfully reconstructed.

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