Fundamentals of Thermal-Fluid Sciences, 3rd Edition
Yunus A. Cengel, Robert H. Turner, John M. Cimbala
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Chapter 8
ENTROPY

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Objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

ENTROPY
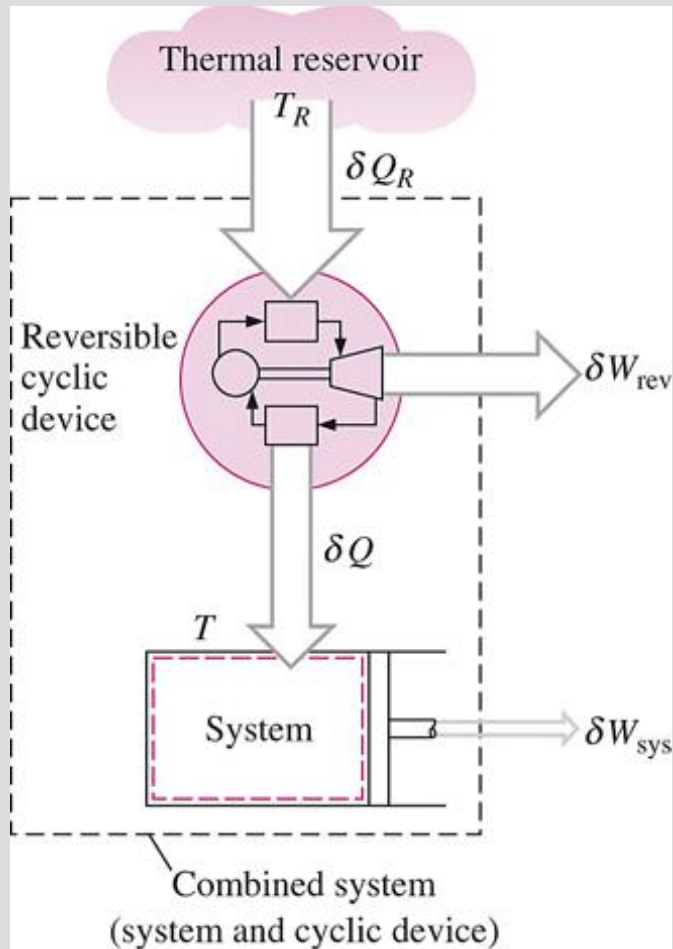
The system considered in the development of the Clausius inequality.

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T} \qquad \delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

$$W_C = T_R \oint \frac{\delta Q}{T}$$

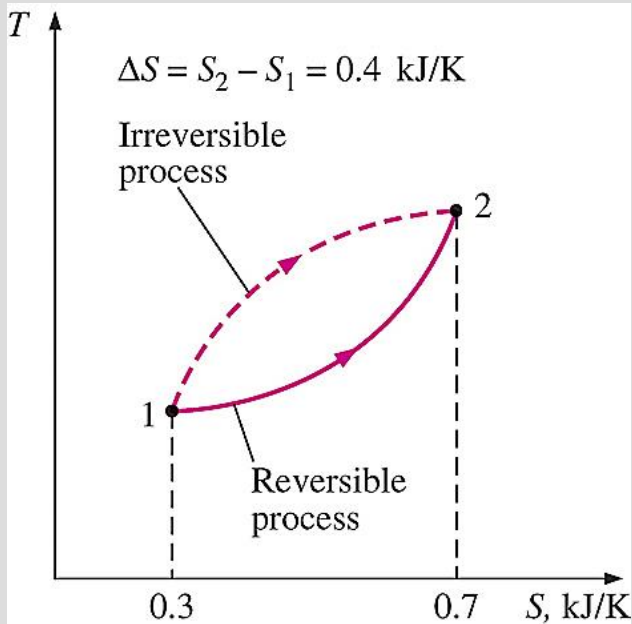$$\oint \frac{\delta Q}{T} \leq 0 \qquad \text{Clasius inequality}$$

$$\oint \left(\frac{\delta Q}{T} \right)_{int\ rev} = 0$$

$$dS = \left(\frac{\delta Q}{T} \right)_{int\ rev} \qquad (kJ/K) \qquad \text{Formal definition of entropy}$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{int\ rev}$$

The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.

$$\Delta S = S_2 - S_1 = 0.4 \text{ kJ/K}$$

Irreversible process

2

1

Reversible process

0.3   0.7 S, kJ/K

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

A quantity whose cyclic integral is zero (i.e., a property like volume)

Entropy is an extensive property of a system.

The entropy change between two specified states is the same whether the process is reversible or irreversible.

1 m³

3 m³

1 m³

The net change in volume (a property) during a cycle is always zero.

$$\oint dV = \Delta V_{\text{cycle}} = 0$$

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}} \qquad \Delta S = \frac{Q}{T_0}$$
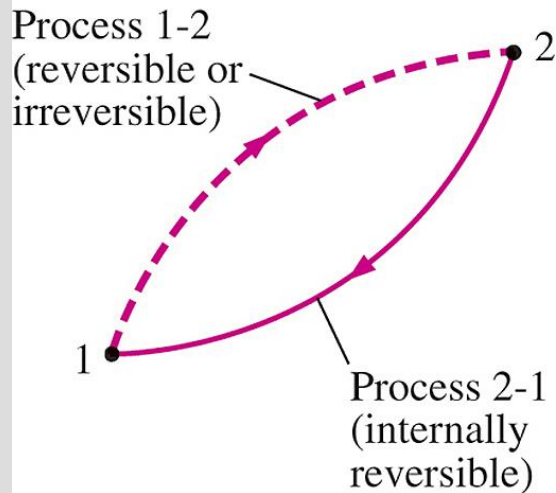
This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

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THE INCREASE OF ENTROPY PRINCIPLE

Process 1-2 (reversible or irreversible)

Process 2-1 (internally reversible)

1

2
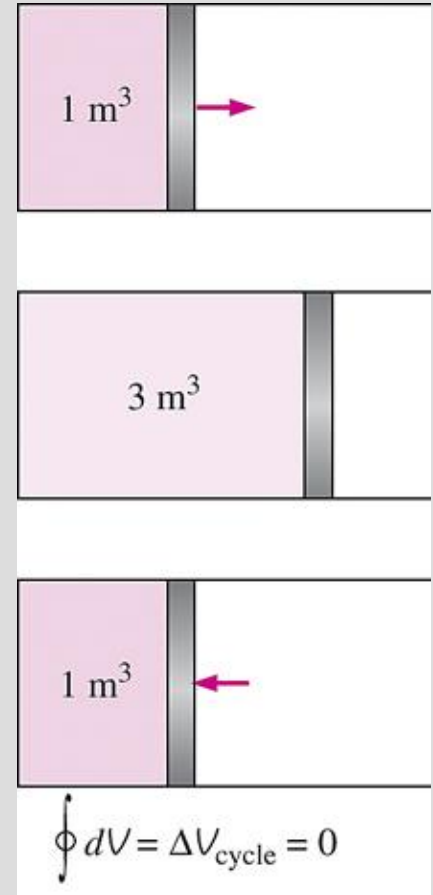
A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0$$

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

$$dS \geq \frac{\delta Q}{T}$$

The equality holds for an internally reversible process and the inequality for an irreversible process.

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{gen} is always a *positive* quantity or zero.
Can the entropy of a system during a process decrease?

(Isolated)

Subsystem 1

Subsystem 2

Subsystem 3

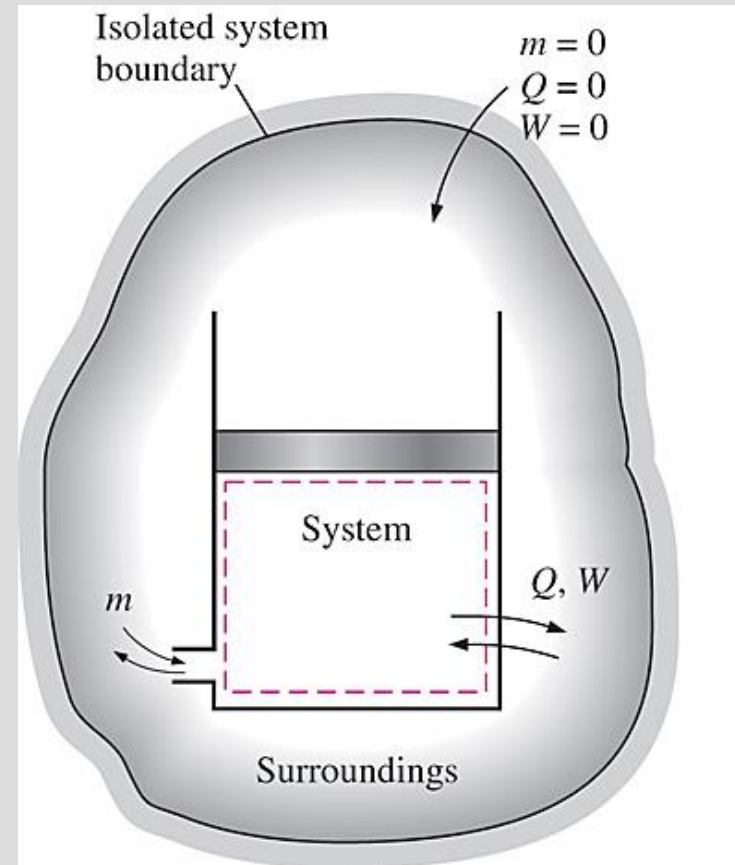Subsystem N

$$\Delta S_{total} = \sum_{i=1}^{N} \Delta S_i > 0$$
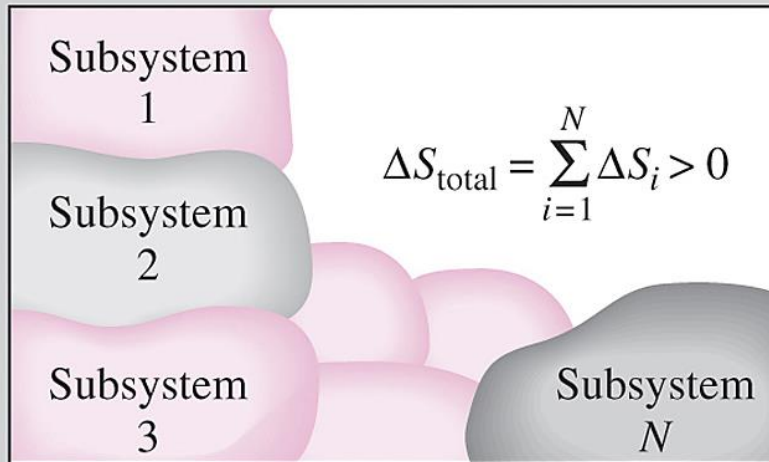
The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{isolated} \geq 0$$

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \geq 0$$
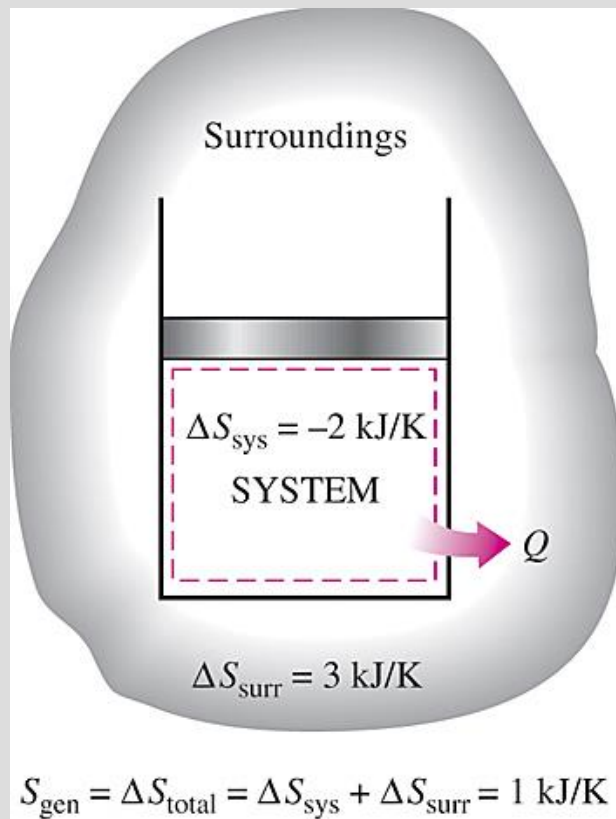
$$S_{gen} \begin{cases} > & 0 \text{ Irreversible process} \\ = & 0 \text{ Reversible process} \\ < & 0 \text{ Impossible process} \end{cases}$$

The increase of entropy principle

Isolated system boundary

$m = 0$
$Q = 0$
$W = 0$

System

Q, W

m

Surroundings

A system and its surroundings form an isolated system.
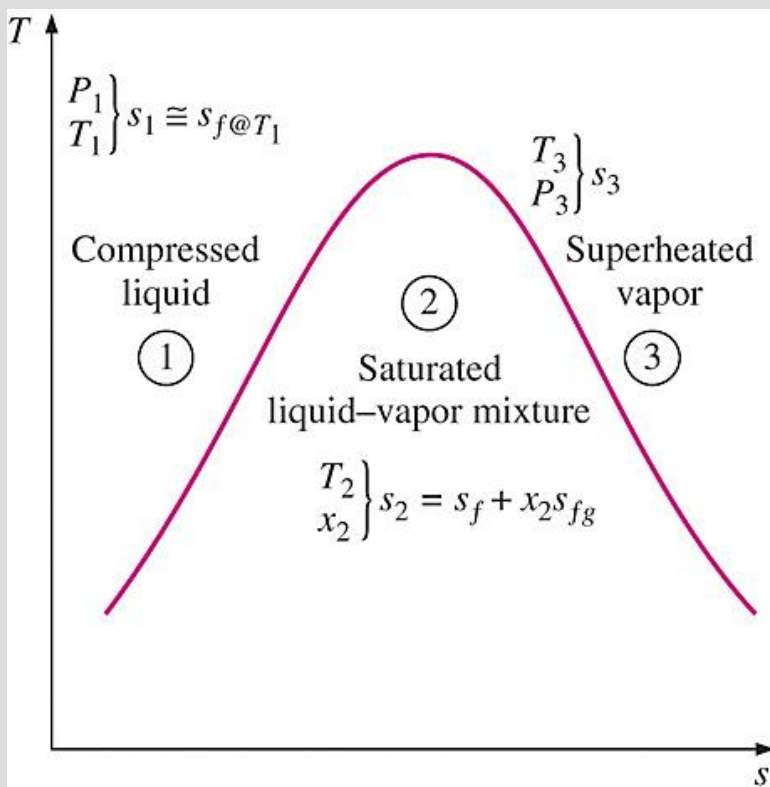
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Some Remarks about Entropy

The entropy change of a system can be negative, but the entropy generation cannot.
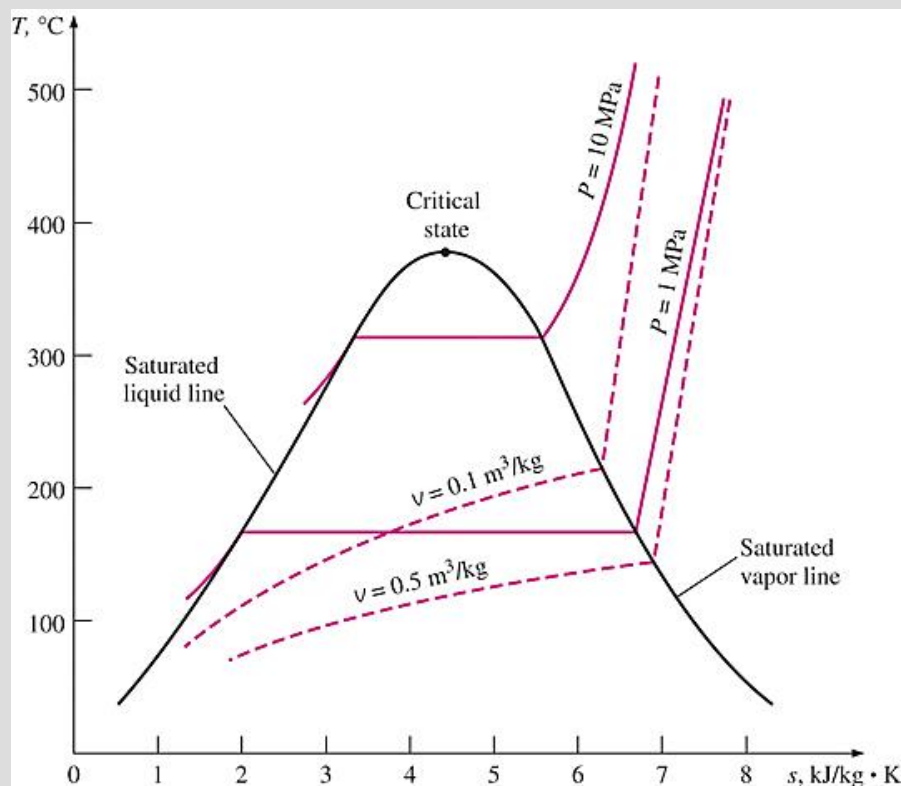
1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{gen} \geq 0$. A process that violates this principle is impossible.

2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.

3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.

Schematic of the *T-s* diagram for water.

The entropy of a pure substance is determined from the tables (like other properties).
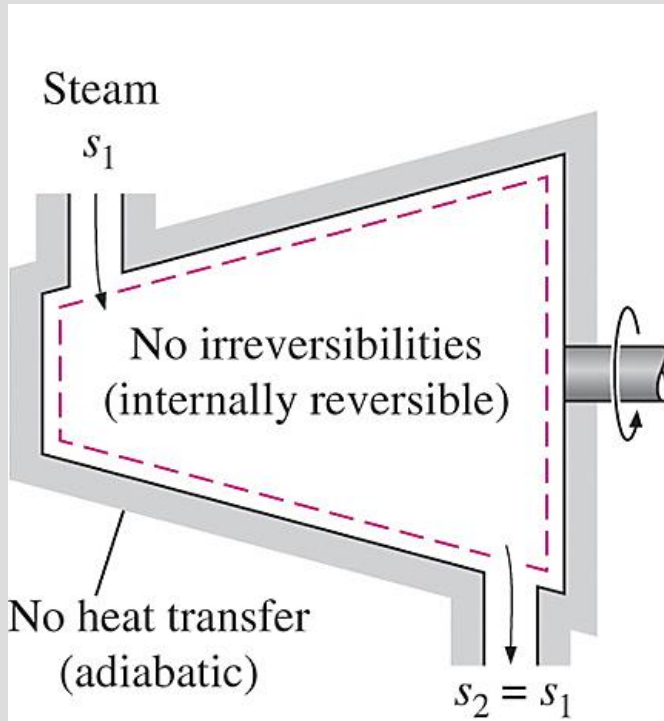
Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \qquad (\text{kJ/K})$$

ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \;\; (\text{kJ/kg} \cdot \text{K})$$

Steam
s_1

No irreversibilities
(internally reversible)

No heat transfer
(adiabatic)

$s_2 = s_1$

During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

T

1

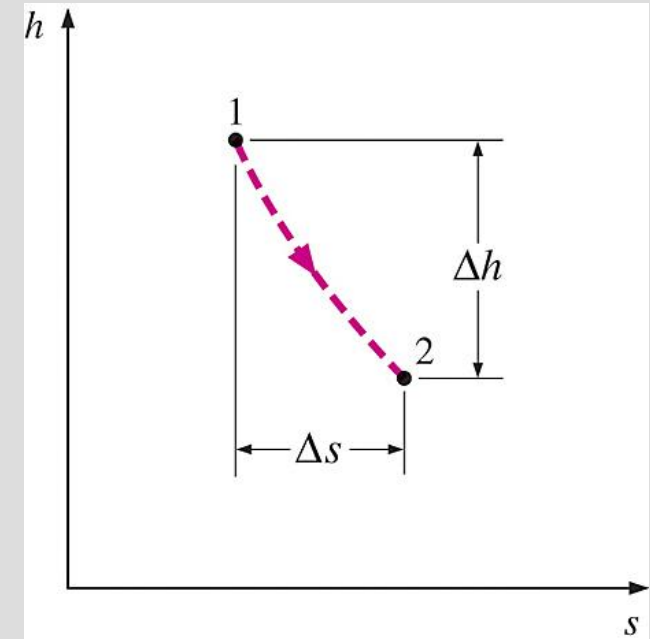Isentropic process

2

$s_2 = s_1$

s
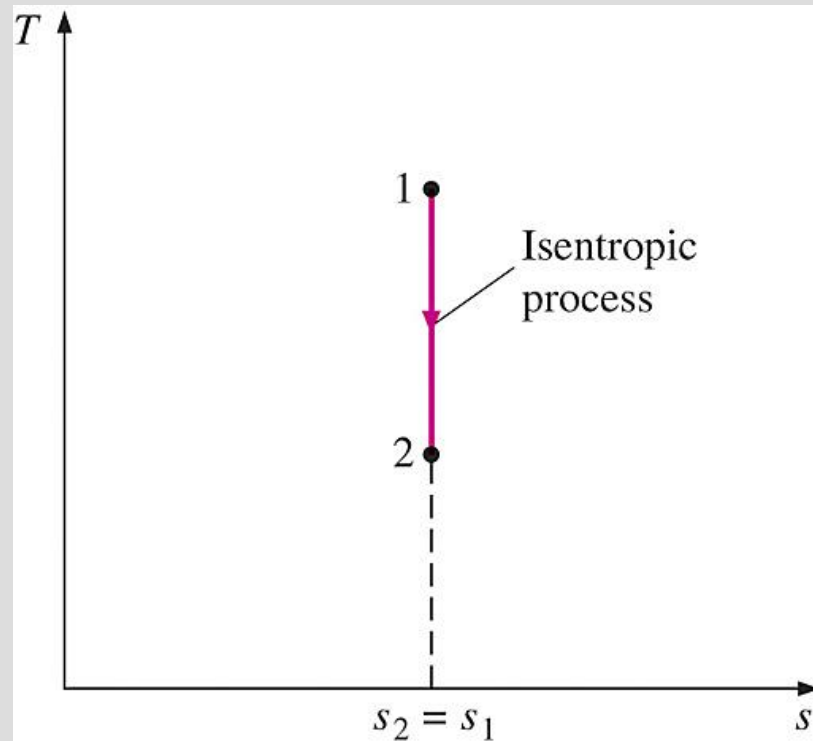
The isentropic process appears as a *vertical* line segment on a *T-s* diagram.

PROPERTY DIAGRAMS INVOLVING ENTROPY
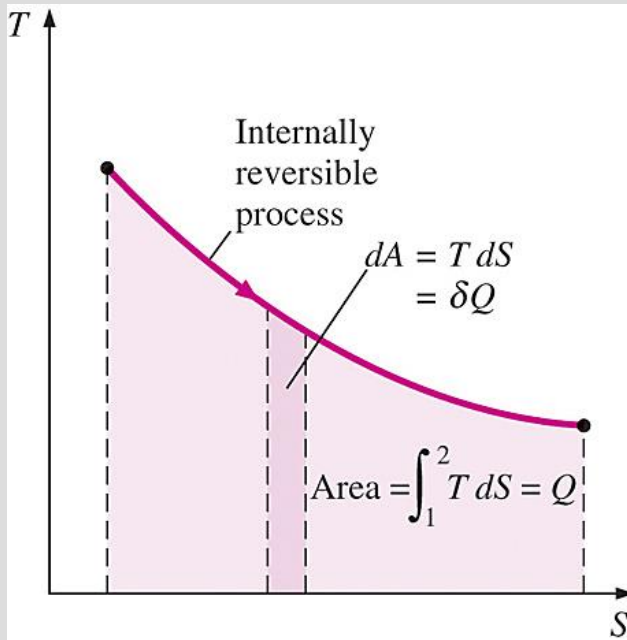
On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible processes.

$$\delta Q_{int\,rev} = T\,dS$$

$$Q_{int\,rev} = \int_1^2 T\,dS$$

$$\delta q_{int\,rev} = T\,ds$$

$$q_{int\,rev} = \int_1^2 T\,ds$$

$$Q_{int\,rev} = T_0\,\Delta S$$

$$q_{int\,rev} = T_0\,\Delta s$$

For adiabatic steady-flow devices, the vertical distance Δh on an *h-s* diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

Mollier diagram: The *h-s* diagram

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WHAT IS ENTROPY?

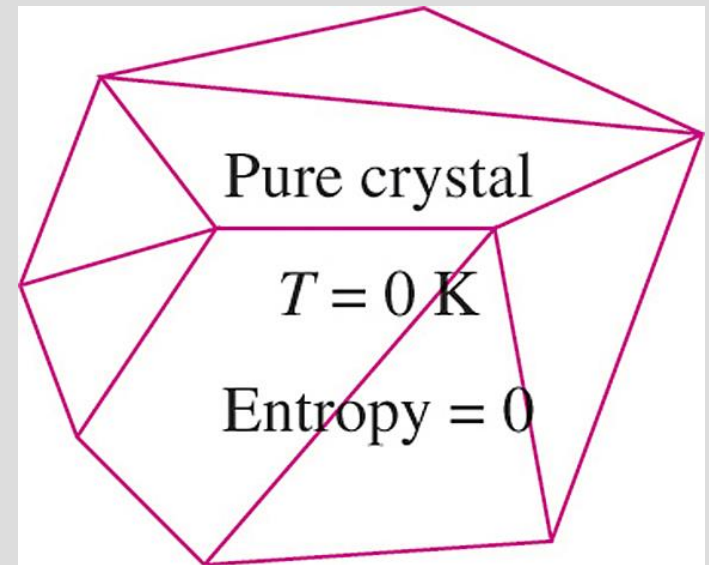Entropy,
kJ/kg · K

GAS

LIQUID

SOLID
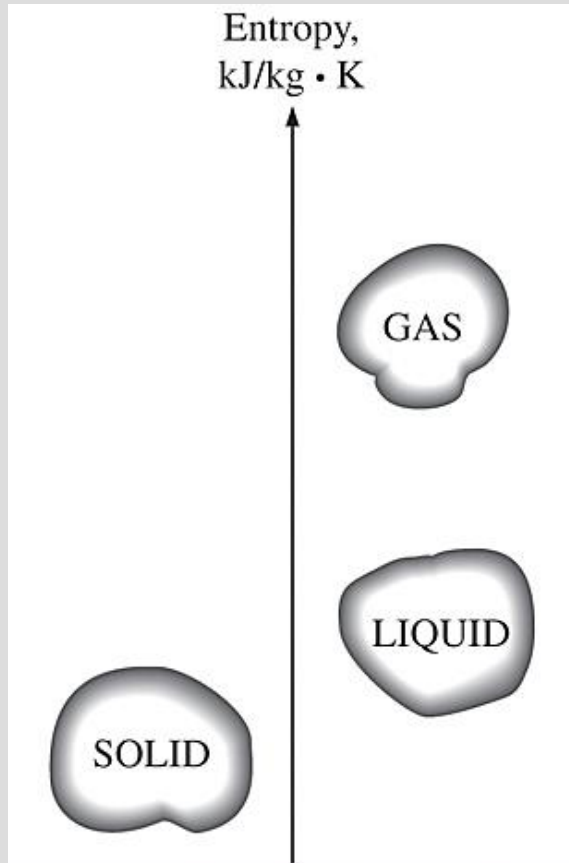
The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Boltzmann relation

$$S = k \ln p$$

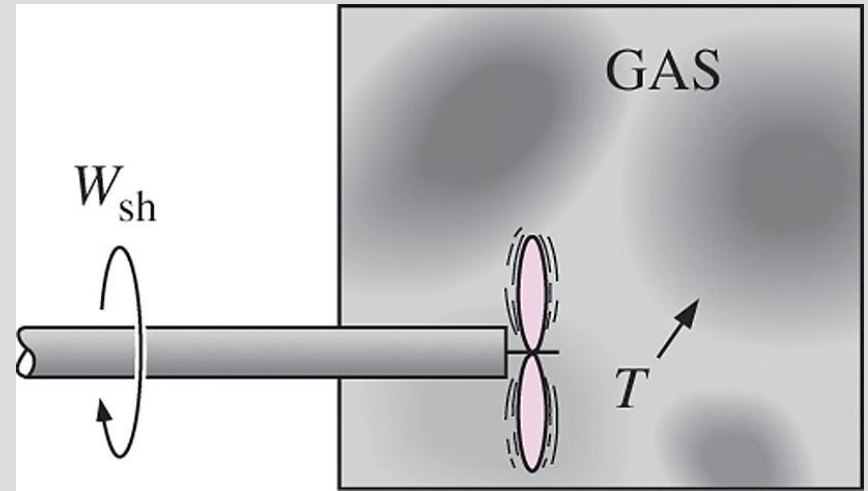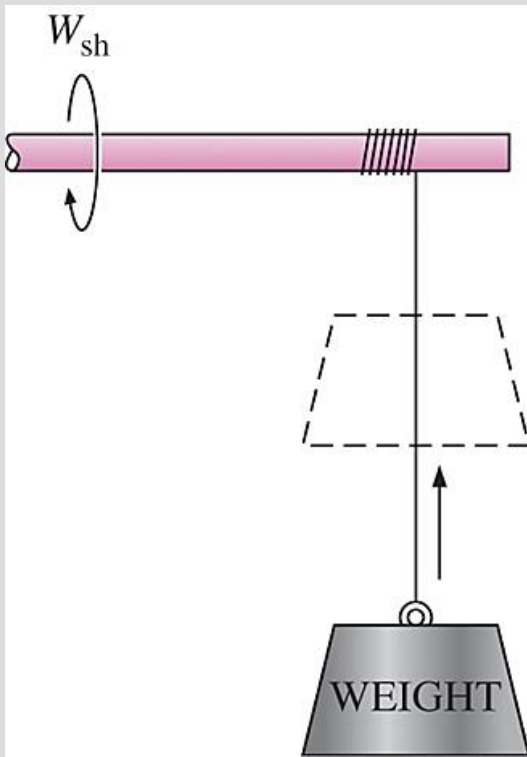$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

Pure crystal

$$T = 0 \text{ K}$$

Entropy = 0

A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (**the third law of thermodynamics**).

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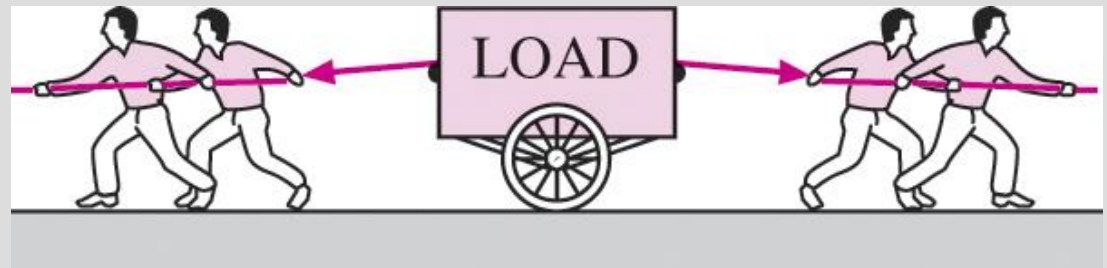
Disorganized energy does not create much useful effect, no matter how large it is.

The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.

HOT BODY

80°C

(Entropy decreases)

Heat

COLD BODY

20°C
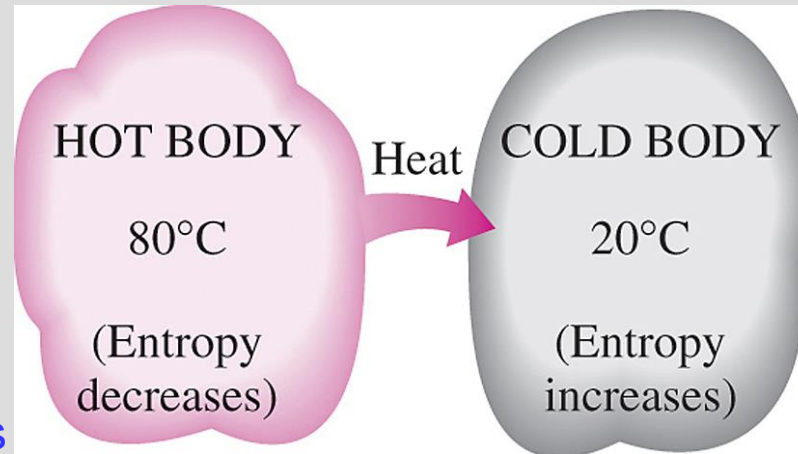
(Entropy increases)

During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE *T ds* RELATIONS

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T\, dS$$

$$\delta W_{\text{int rev,out}} = P\, dV$$

$$T\, dS = dU + P\, dV \qquad (\text{kJ})$$

$$T\, ds = du + P\, dv \qquad (\text{kJ/kg})$$

the first *T ds*, or *Gibbs equation*

$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P\, dv + v\, dP \\ T\, ds = du + P\, dv \end{array} \right\} \quad T\, ds = dh - v\, dP$$

the second *T ds equation*
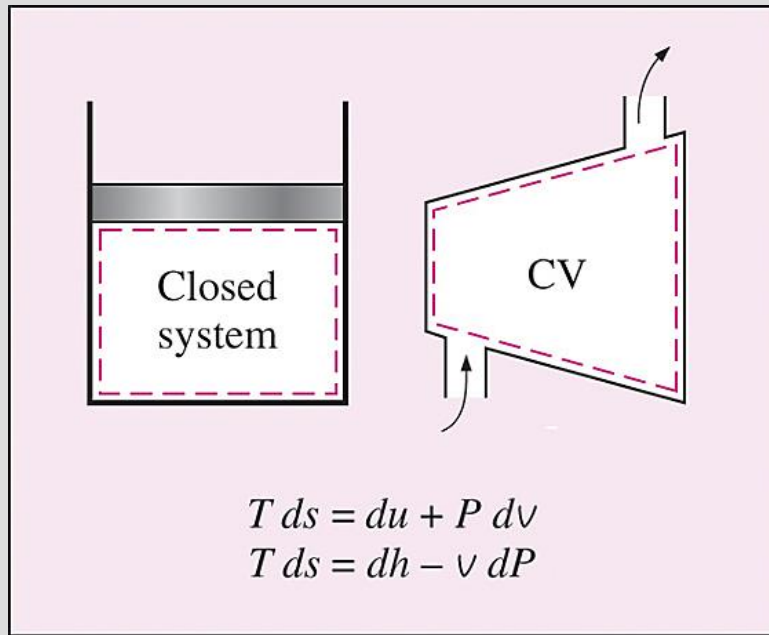
The *T ds* relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$ds = \frac{du}{T} + \frac{P\, dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v\, dP}{T}$$

Differential changes in entropy in terms of other properties

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ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P \; dv}{T}$$

Since $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c \; dT}{T}$$

since $c_p = c_v = c$ and $du = c \; dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

Liquids, solids: $$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{avg} \ln \frac{T_2}{T_1} \qquad (kJ/kg \cdot K)$$

For and isentropic process of an incompressible substance

Isentropic: $$s_2 - s_1 = c_{avg} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

THE ENTROPY CHANGE OF IDEAL GASES

From the first *T ds* relation

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

$$du = c_v\,dT$$

$$P = RT/v$$

$$ds = c_v\,\frac{dT}{T} + R\,\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\,\ln\frac{v_2}{v_1}$$

From the second *T ds* relation

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

$$dh = c_p\,dT \qquad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T)\,\frac{dT}{T} - R\,\ln\frac{P_2}{P_1}$$

$$Pv = RT$$
$$du = C_v\,dT$$
$$dh = C_p\,dT$$

A broadcast from channel IG.

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Constant Specific Heats (Approximate Analysis)

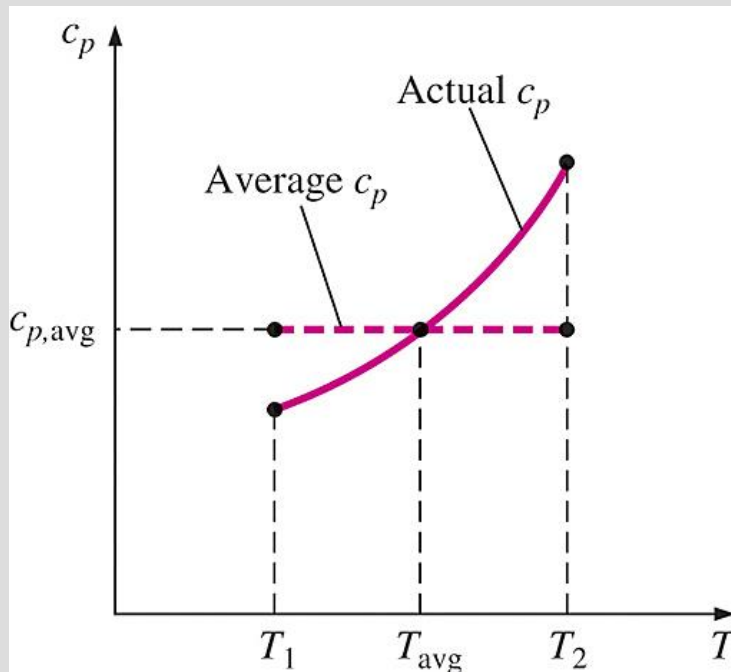$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1} \quad \longrightarrow \quad s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1} \quad \longrightarrow \quad s_2 - s_1 = c_{p,avg} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$(\text{kJ/kg} \cdot \text{K})$$

Entropy change of an ideal gas on a unit–mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,avg} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \qquad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,avg} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \qquad (\text{kJ/kmol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

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Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function $s°$ as

$$s° = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2° - s_1°$$

T, K	$s°$, kJ/kg $\cdot$ K
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

(Table A-17)

On a unit–mass basis

$$s_2 - s_1 = s_2° - s_1° - R \ln \frac{P_2}{P_1} \qquad (\text{kJ/kg} \cdot \text{K})$$

On a unit–mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2° - \bar{s}_1° - R_u \ln \frac{P_2}{P_1} \qquad (\text{kJ/kmol} \cdot \text{K})$$

The entropy of an ideal gas depends on both T and P. The function s represents only the temperature-dependent part of entropy.

Isentropic Processes of Ideal Gases
Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, \quad k = c_p/c_v$$
and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^{k}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

VALID FOR *ideal gas
  *isentropic process
  *constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$Tv^{k-1} = \text{constant}$$
$$TP^{(1-k)/k} = \text{constant}$$
$$Pv^{k} = \text{constant}$$

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \longrightarrow s_2^\circ = s_1^\circ + R \ln \frac{P_2}{P_1}$$

Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R}$$

$\exp(s^\circ/R)$ is the relative pressure P_r.

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ /R)}{\exp(s_1^\circ /R)}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

The use of P_r data for calculating the final temperature during an isentropic process.

Process: isentropic
Given: P_1, T_1, and P_2
Find: T_2

T	P_r
.	.
.	.
.	.
$T_2 \xleftarrow{\text{read}}$	$P_{r2} = \frac{P_2}{P_1} P_{r1}$
.	.
$T_1 \xrightarrow{\text{read}}$	P_{r1}
.	.

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1}\frac{P_1}{P_2} = \frac{T_2}{T_1}\frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$
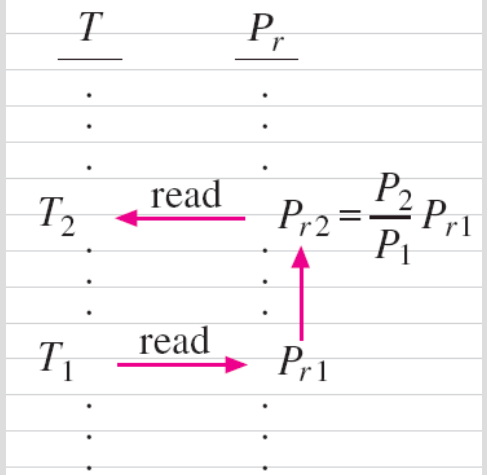
T/P_r is the relative specific volume v_r.

The use of v_r data for calculating the final temperature during an isentropic process

Process: isentropic
Given: v_1, T_1, and v_2
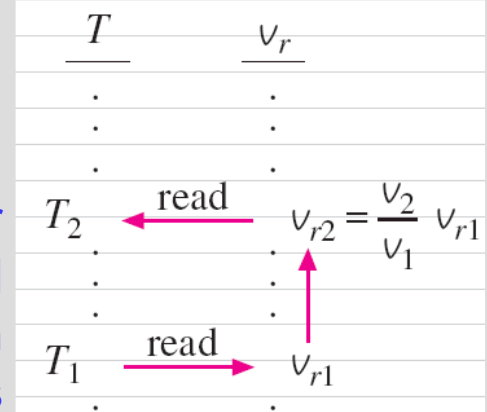Find: T_2

T	v_r
.	.
.	.
.	.
$T_2 \xleftarrow{\text{read}}$	$v_{r2} = \frac{v_2}{v_1} v_{r1}$
.	.
$T_1 \xrightarrow{\text{read}}$	v_{r1}
.	.

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REVERSIBLE STEADY-FLOW WORK

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

$$\delta q_{rev} = T\,ds \qquad (Eq.\ 7–16)$$
$$T\,ds = dh - v\,dP \qquad (Eq.\ 7–24)$$
$$\delta q_{rev} = dh - v\,dP$$

$$-\delta w_{rev} = v\,dP + dke + dpe$$

$$w_{rev} = -\int_1^2 v\,dP - \Delta ke - \Delta pe$$

$$w_{rev} = -\int_1^2 v\,dP$$

When kinetic and potential energies are negligible

$$w_{rev,in} = \int_1^2 v\,dP + \Delta ke + \Delta pe$$
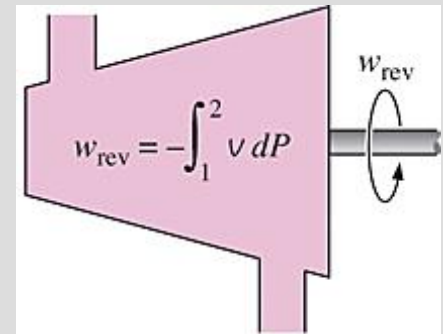
$$w_{rev} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (Bernoulli equation):
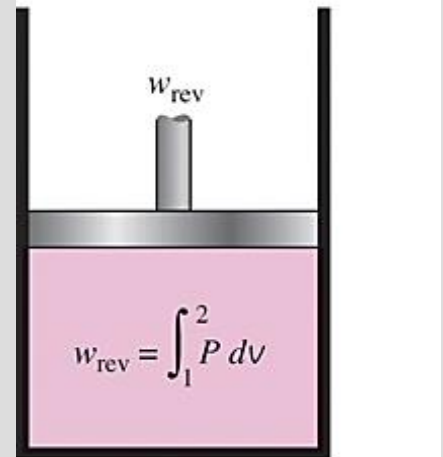
$$v(P_2 - P_1) + \frac{V_2^2 - V_1^1}{2} + g(z_2 - z_1) = 0$$

$$w = -\int_1^2 v\,dP$$

$$W = -\int_1^2 V\,dP$$

$$W = -\int_1^2 V\,dP$$

The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

$$w_{rev} = -\int_1^2 v\,dP$$

(a) Steady-flow system

$$w_{rev} = \int_1^2 P\,dv$$

(b) Closed system

Reversible work relations for steady-flow and closed systems.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{act} - \delta w_{act} = dh + dke + dpe \quad \text{Actual}$$

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe \quad \text{Reversible}$$

$$\delta q_{act} - \delta w_{act} = \delta q_{rev} - \delta w_{rev}$$

$$\delta w_{rev} - \delta w_{act} = \delta q_{rev} - \delta q_{act}$$
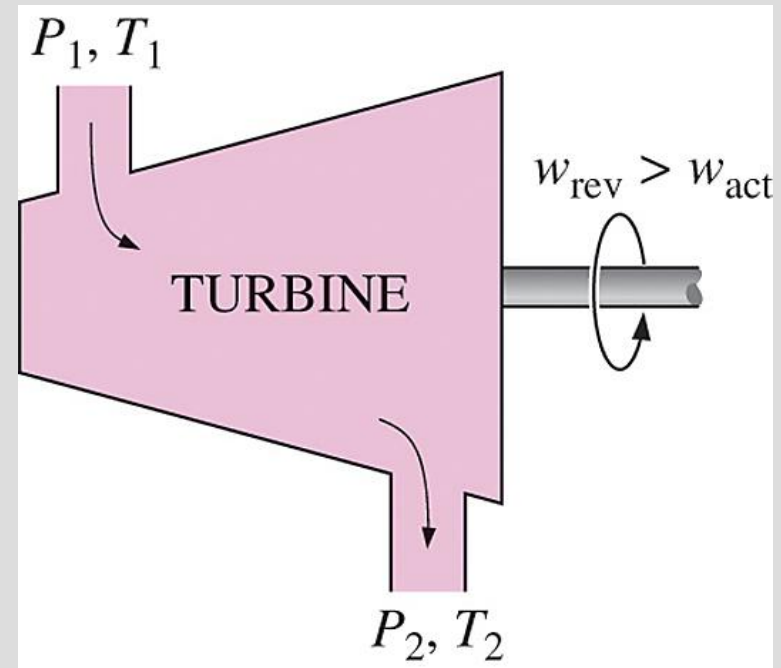
$$\delta q_{rev} = T\,ds \qquad ds \geq \frac{\delta q_{act}}{T}$$

$$\frac{\delta w_{rev} - \delta w_{act}}{T} = ds - \frac{\delta q_{act}}{T} \geq 0$$

$$\partial w_{rev} \geq \partial w_{act}$$

$$w_{rev} \geq w_{act}$$

Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.

P_1, T_1

TURBINE

$w_{rev} > w_{act}$

P_2, T_2

A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

MINIMIZING THE COMPRESSOR WORK

$$w_{rev,in} = \int_1^2 v\, dP$$

When kinetic and potential energies are negligible

Isentropic (Pv^k = constant):

$$w_{comp,in} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

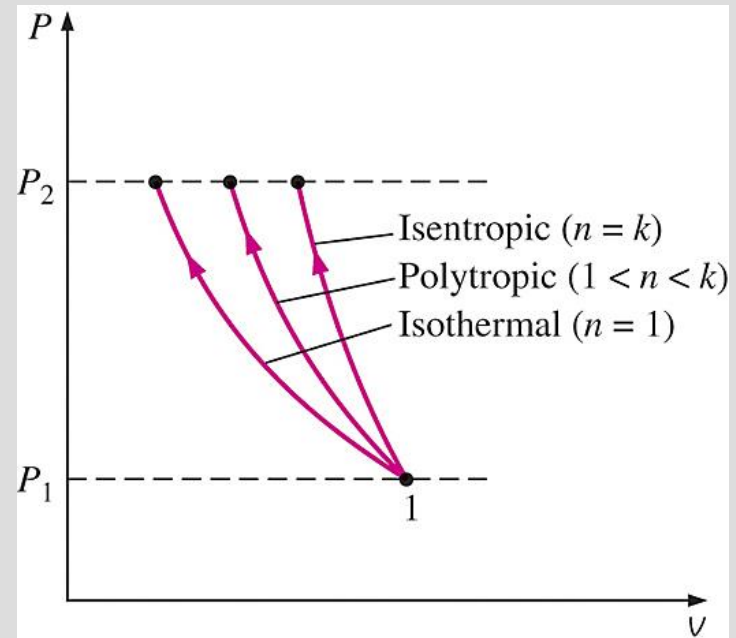Polytropic (Pv^n = constant):

$$w_{comp,in} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Isothermal (Pv = constant):

$$w_{comp,in} = RT \ln \frac{P_2}{P_1}$$

The adiabatic compression (Pv^k = constant) requires the maximum work and the isothermal compression (T = constant) requires the minimum. **Why?**
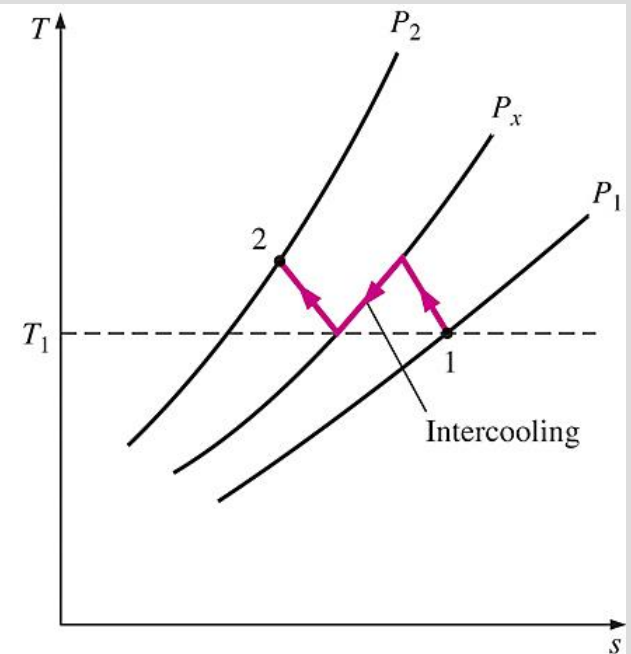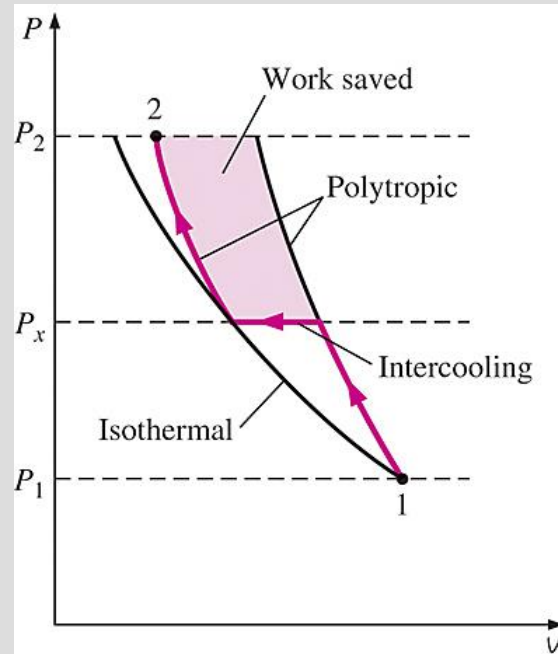

P-v diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

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Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an *intercooler*.

P-v and *T-s* diagrams for a two-stage steady-flow compression process.

$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

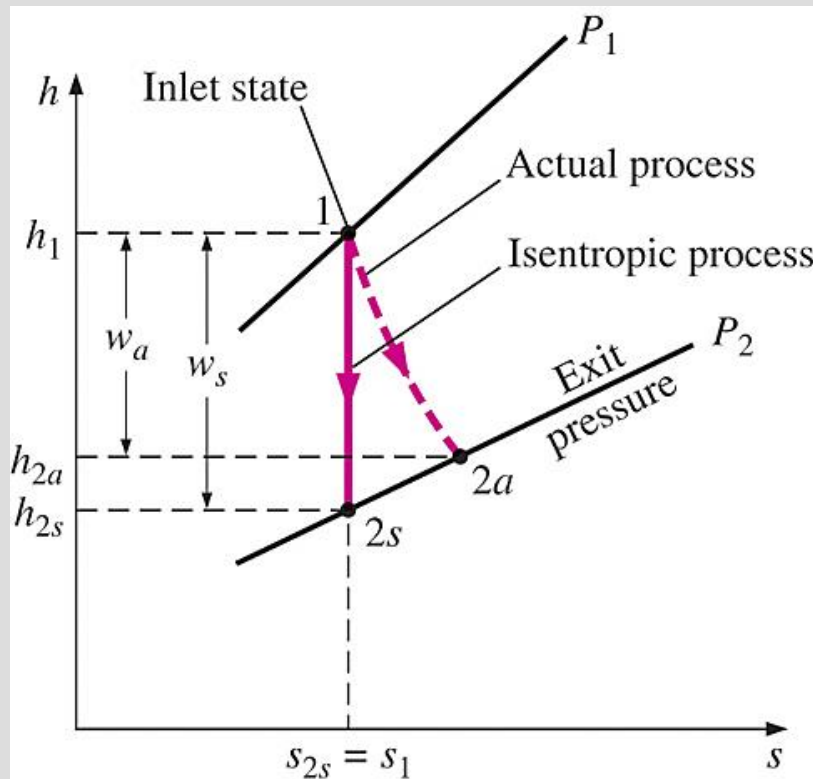$$P_x = (P_1 P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.
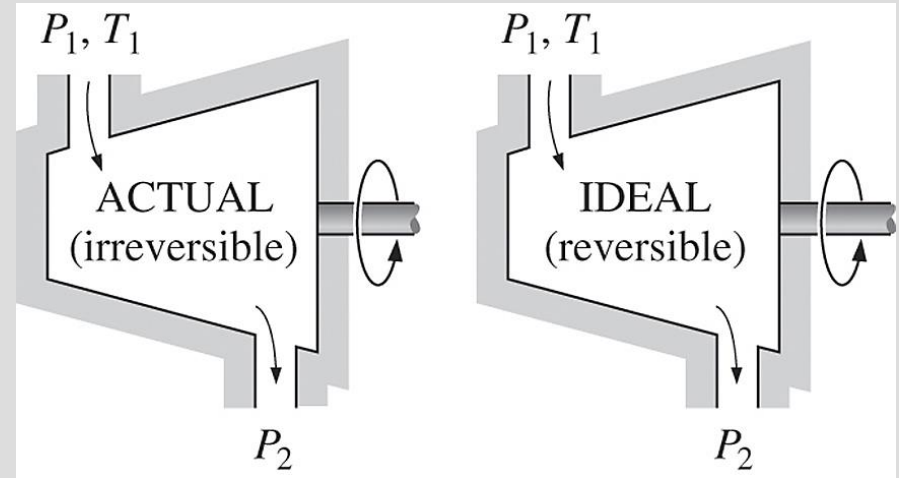
Isentropic Efficiency of Turbines

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine.

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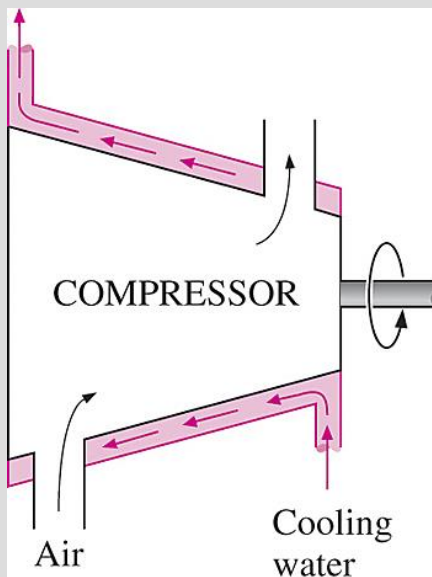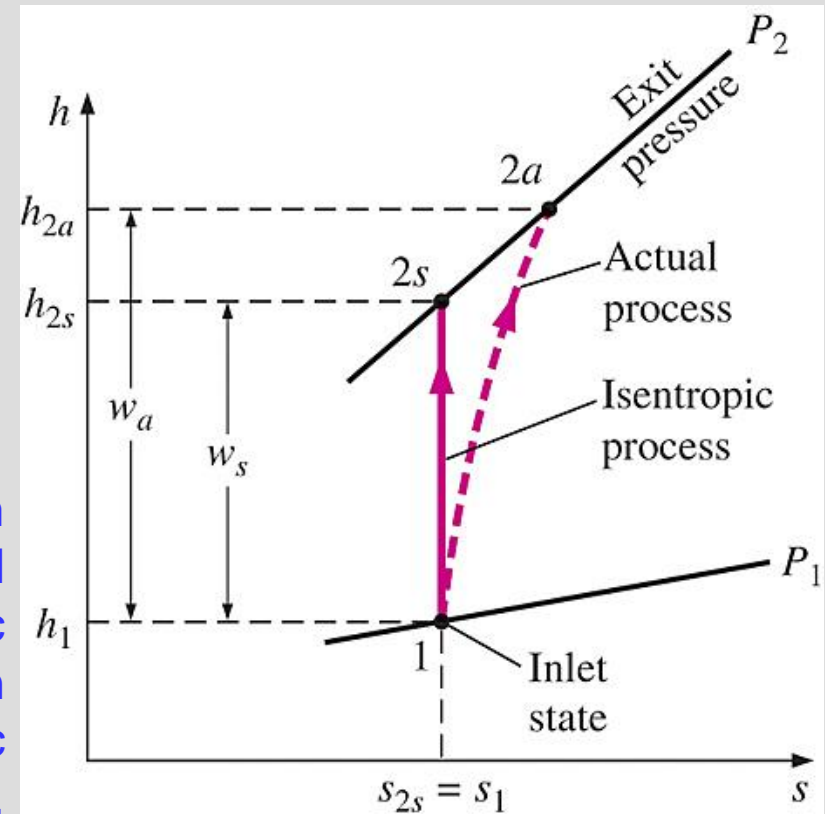# Isentropic Efficiencies of Compressors and Pumps

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

When kinetic and potential energies are negligible

$$\eta_P = \frac{w_s}{w_a} = \frac{\vee(P_2 - P_1)}{h_{2a} - h_1}$$

For a pump

$$\eta_C = \frac{w_t}{w_a}$$

Isothermal efficiency

The *h-s* diagram of the actual and isentropic processes of an adiabatic compressor.

Compressors are sometimes intentionally cooled to minimize the work input.

Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

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Isentropic Efficiency of Nozzles

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

Then,

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle.
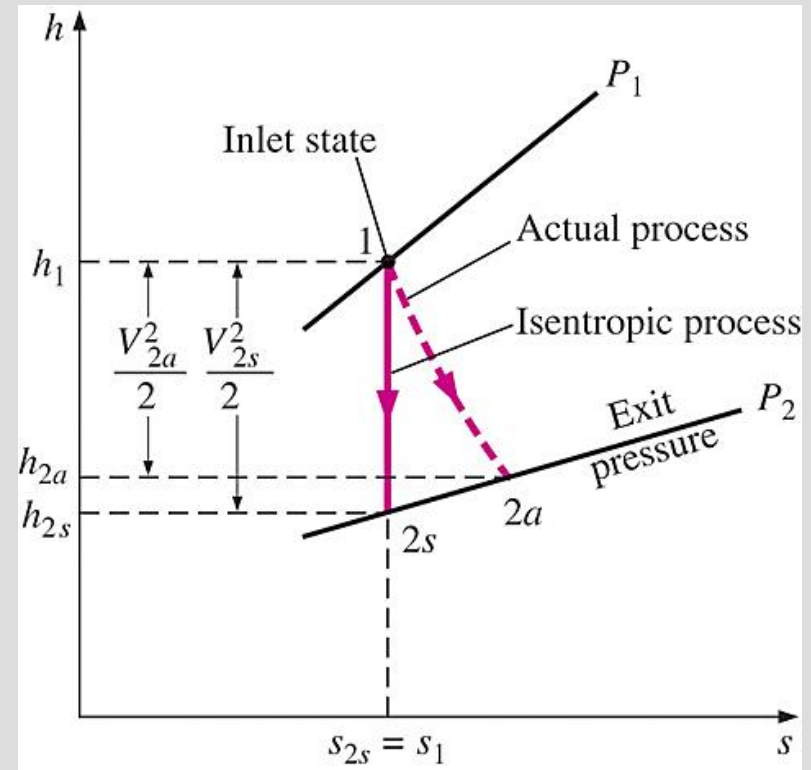
A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.

| 950 K AIR | Actual nozzle ⟶ 764 K, 639 m/s |
| | Isentropic nozzle ⟶ 748 K, 666 m/s |
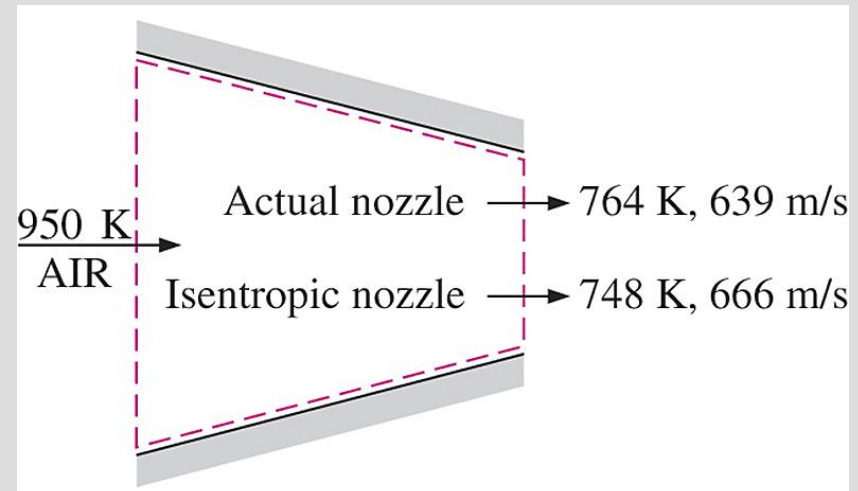
ENTROPY BALANCE

$$\begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{entering} \end{pmatrix} - \begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{pmatrix} + \begin{pmatrix} \text{Total} \\ \text{entropy} \\ \text{generated} \end{pmatrix} = \begin{pmatrix} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{pmatrix}$$

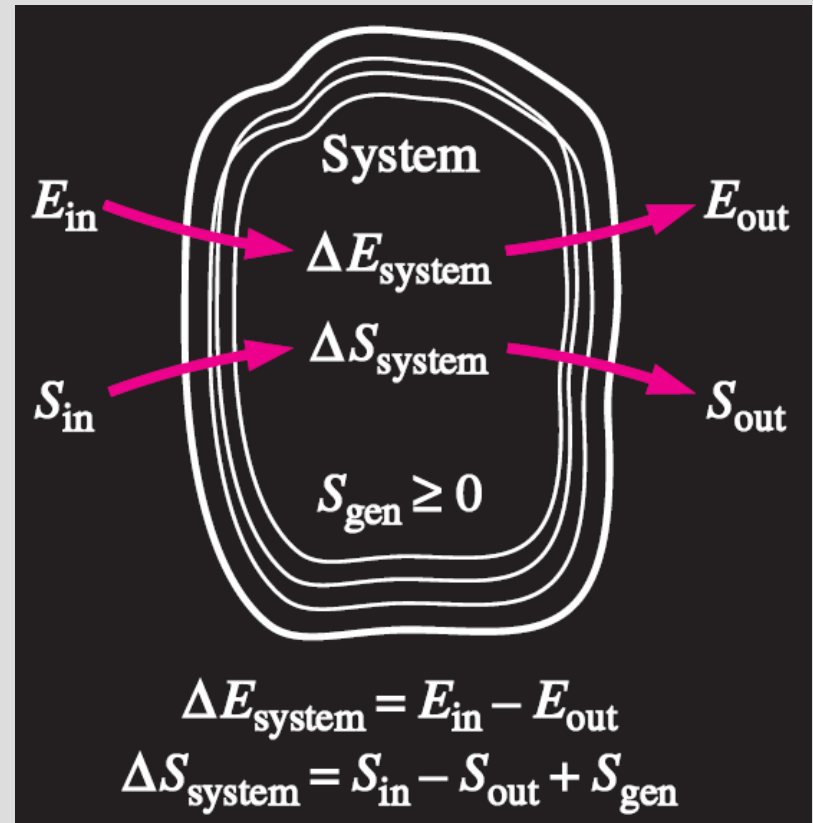$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

Entropy Change of a System, ΔS_{system}

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

$$S_{\text{system}} = \int s\, \delta m = \int_V s\rho\, dV$$

Energy and entropy balances for a system.

System

E_{in} ΔE_{system} E_{out}

S_{in} ΔS_{system} S_{out}

$S_{\text{gen}} \geq 0$

$$\Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}}$$
$$\Delta S_{\text{system}} = S_{\text{in}} - S_{\text{out}} + S_{\text{gen}}$$

Mechanisms of Entropy Transfer, S_{in} and S_{out}

1 Heat Transfer
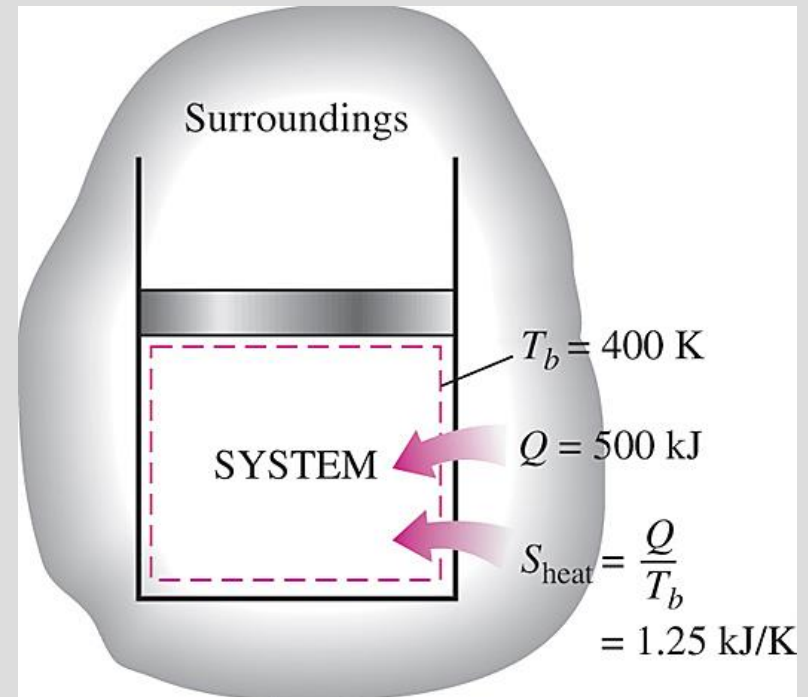
Entropy transfer by heat transfer:
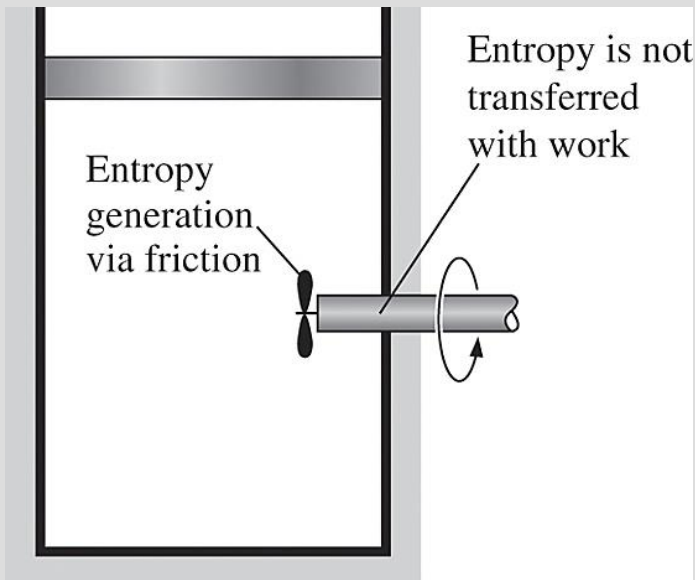
$$S_{heat} = \frac{Q}{T} \qquad (T = \text{constant})$$

$$S_{heat} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

Entropy transfer by work:

$$S_{work} = 0$$

Entropy
generation
via friction

Entropy is not
transferred
with work

Surroundings

$T_b = 400$ K

SYSTEM

$Q = 500$ kJ

$S_{heat} = \dfrac{Q}{T_b}$

$= 1.25$ kJ/K

Heat transfer is always accompanied by entropy transfer in the amount of Q/T, where T is the boundary temperature.

No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

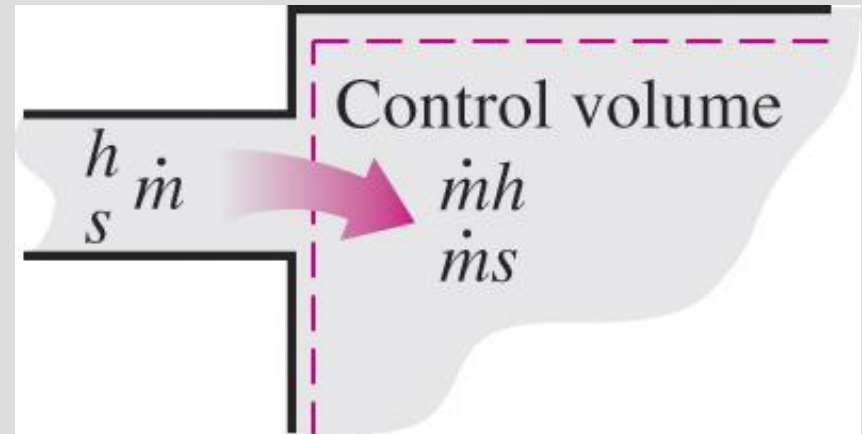# Mechanisms of Entropy Transfer, S_{in} and S_{out}

2 Mass Flow

Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s\rho V_n \, dA_c$$

$$S_{\text{mass}} = \int s \, \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} \, dt$$

Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

Entropy Generation, S_{gen}

$$\underbrace{S_{in} - S_{out}}_{\substack{\text{Net entropy transfer} \\ \text{by heat and mass}}} + \underbrace{S_{gen}}_{\substack{\text{Entropy} \\ \text{generation}}} = \underbrace{\Delta S_{system}}_{\substack{\text{Change} \\ \text{in entropy}}} \quad (kJ/K)$$

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy} \\ \text{transfer by heat} \\ \text{and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{dS_{system}/dt}_{\substack{\text{Rate of change} \\ \text{in entropy}}} \quad (kW/K)$$

$$(s_{in} - s_{out}) + s_{gen} = \Delta s_{system} \quad (kJ/kg \cdot K)$$

Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.
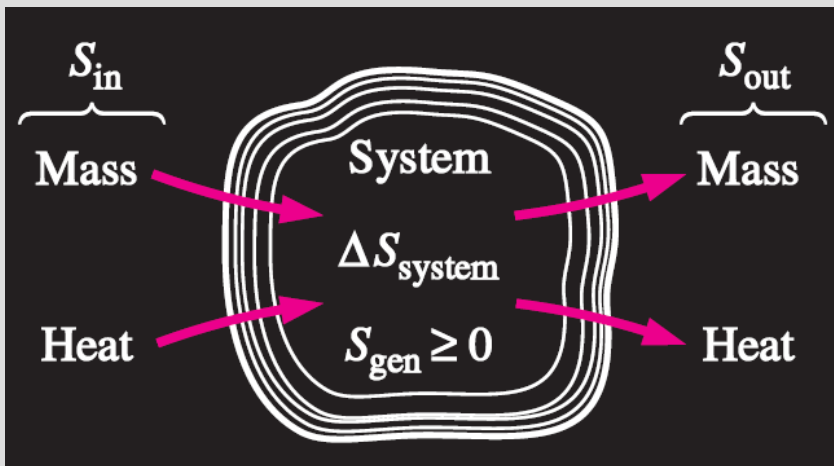
Mechanisms of entropy transfer for a general system.
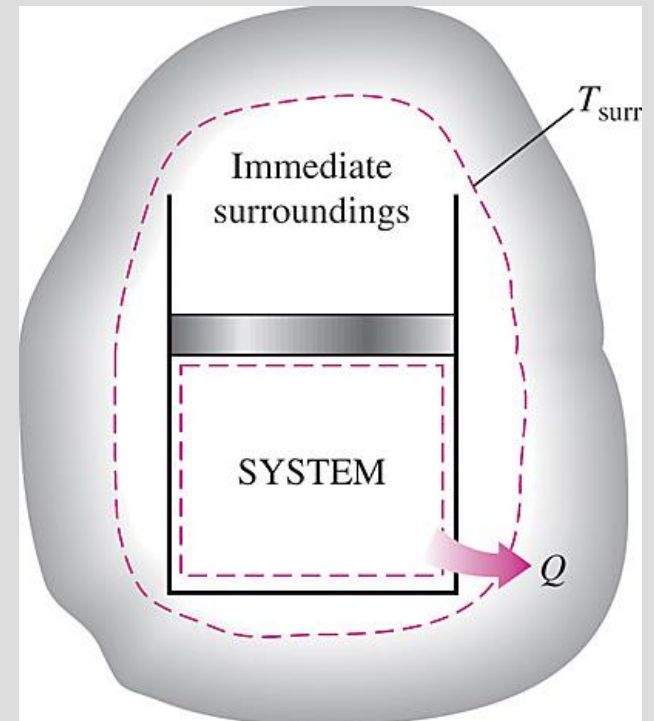
Closed Systems

Closed system: $\qquad \sum \dfrac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \qquad \text{(kJ/K)}$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

Adiabatic closed system: $\qquad S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$

System + Surroundings: $\qquad S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$

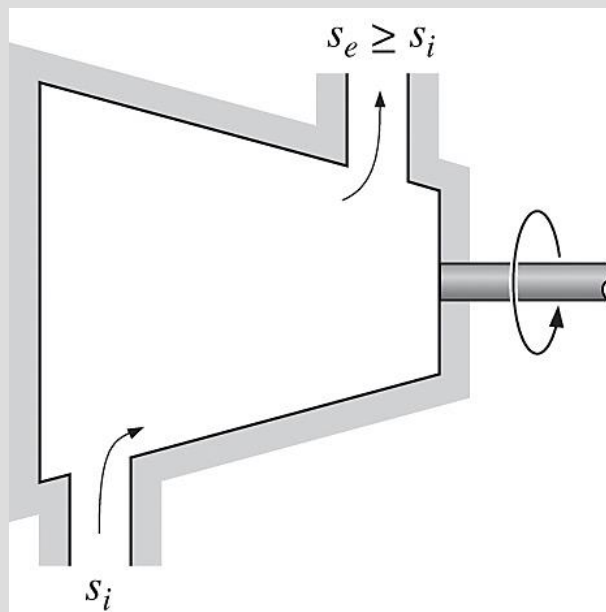$\Delta S_{\text{system}} = m(s_2 - s_1) \qquad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$

Control Volumes

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{gen} = (S_2 - S_1)_{CV} \qquad (kJ/K)$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen} = dS_{CV}/dt \qquad (kW/K)$$

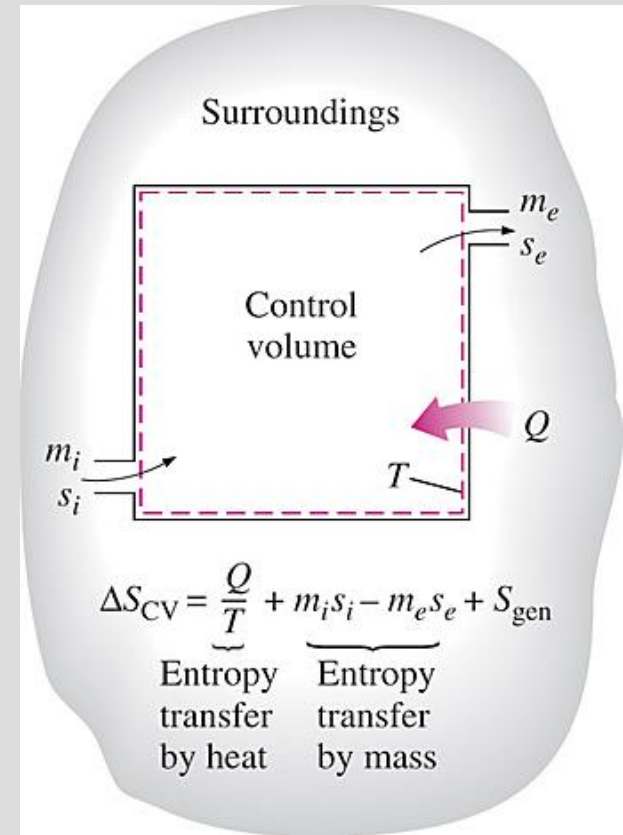*Steady-flow:* $\qquad \dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$

Steady-flow, single-stream: $\qquad \dot{S}_{gen} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$

Steady-flow, single-stream, adiabatic: $\qquad \dot{S}_{gen} = \dot{m}(s_e - s_i)$

$s_e \geq s_i$

The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

Surroundings

Control volume

m_e
s_e

m_i
s_i

Q

T

$$\Delta S_{CV} = \underbrace{\frac{Q}{T}}_{\substack{\text{Entropy} \\ \text{transfer} \\ \text{by heat}}} + \underbrace{m_i s_i - m_e s_e}_{\substack{\text{Entropy} \\ \text{transfer} \\ \text{by mass}}} + S_{gen}$$
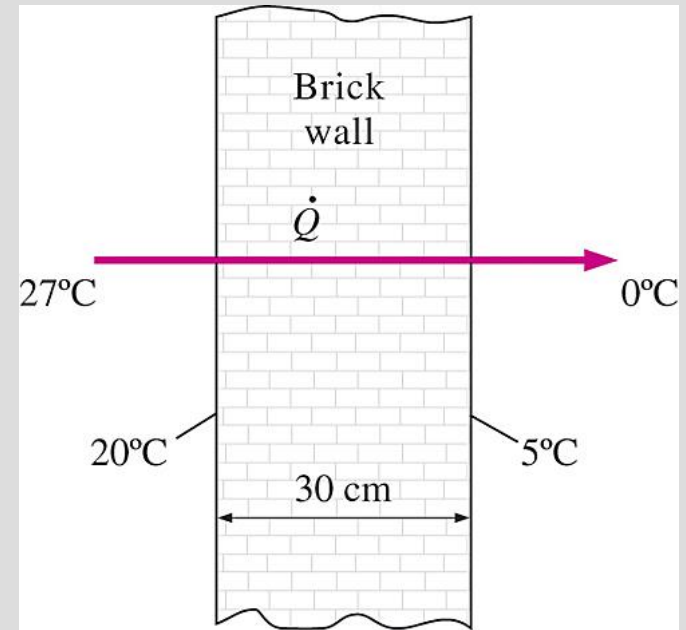
The entropy of a control volume changes as a result of mass flow as well as heat transfer.

EXAMPLES

Entropy balance for heat transfer through a wall

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy} \\ \text{transfer by heat} \\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{dS_{\text{system}}^{\nearrow 0\,(\text{steady})}/dt}_{\substack{\text{Rate of change} \\ \text{in entropy}}}$$

$$\left(\frac{\dot{Q}}{T}\right)_{\text{in}} - \left(\frac{\dot{Q}}{T}\right)_{\text{out}} + \dot{S}_{\text{gen}} = 0$$

Entropy balance for a throttling process

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy} \\ \text{transfer by heat} \\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{dS_{\text{system}}^{\nearrow 0\,(\text{steady})}/dt}_{\substack{\text{Rate of change} \\ \text{in entropy}}}$$

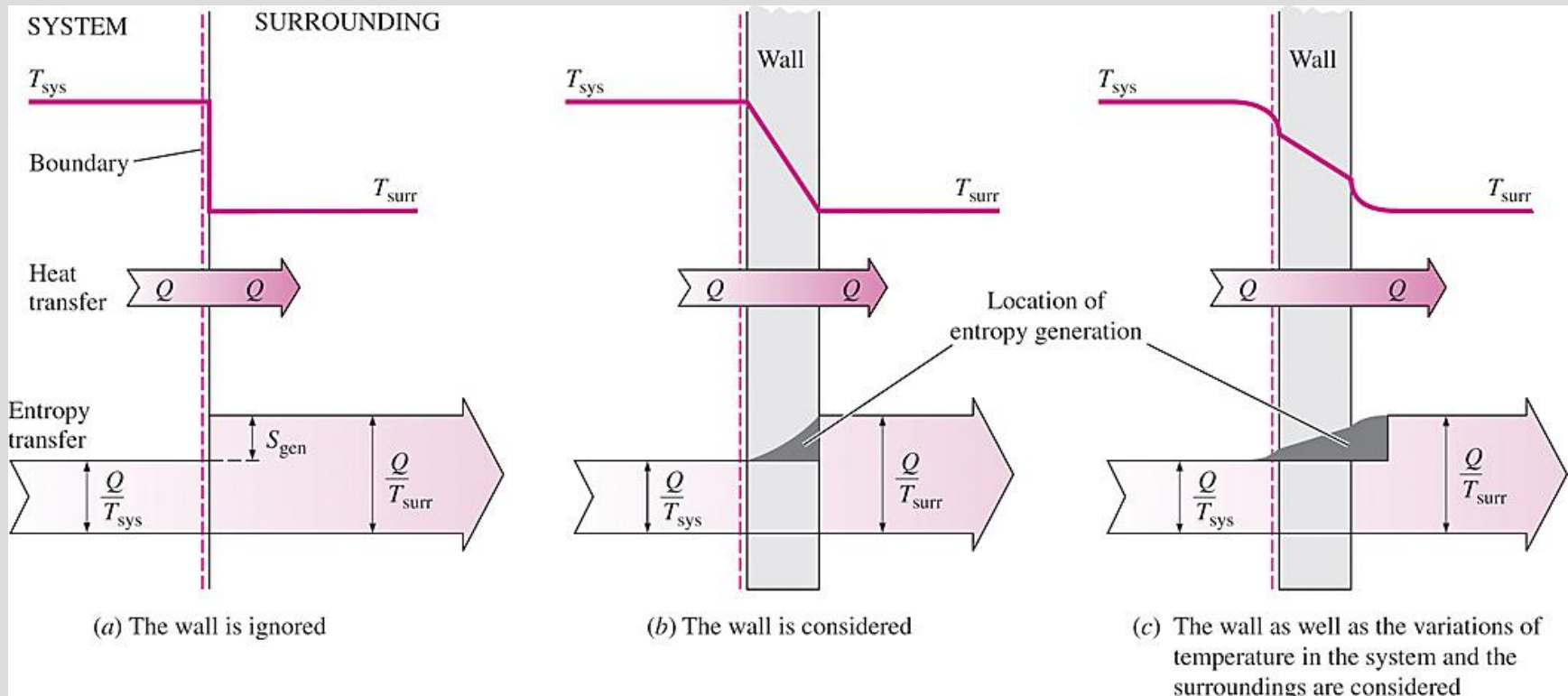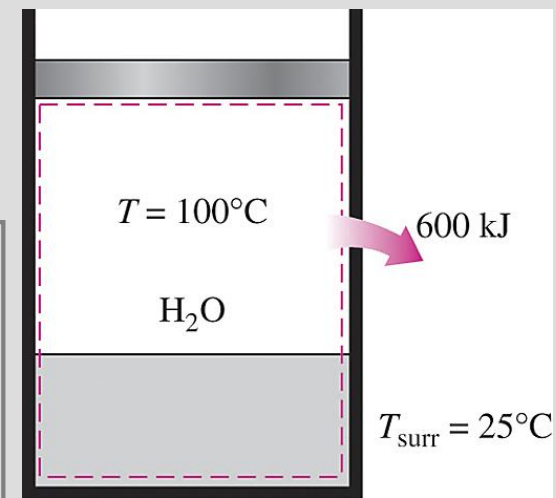$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$
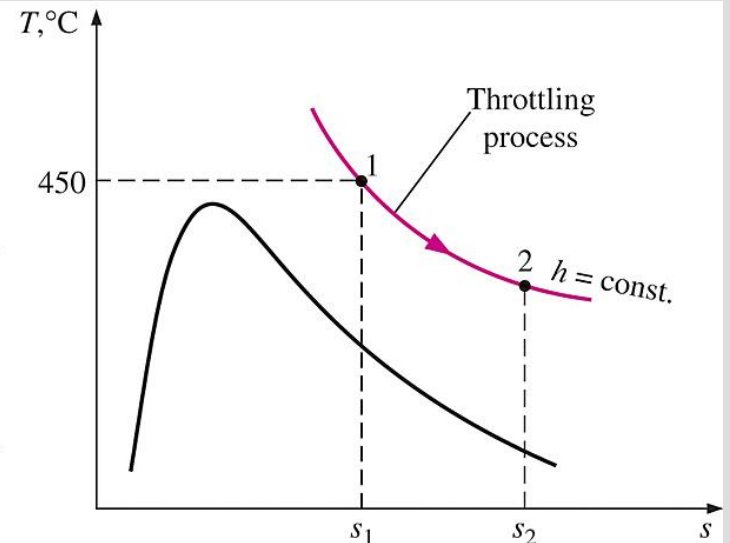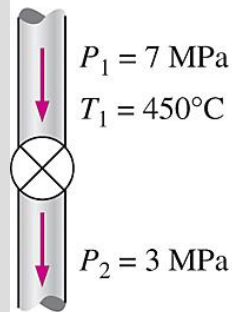
Entropy generation associated with a heat transfer process

$$\underbrace{S_{in} - S_{out}}_{\substack{\text{Net entropy transfer} \\ \text{by heat and mass}}} + \underbrace{S_{gen}}_{\substack{\text{Entropy} \\ \text{generation}}} = \underbrace{\Delta S_{system}}_{\substack{\text{Change} \\ \text{in entropy}}}$$

$$-\frac{Q_{out}}{T_b} + S_{gen} = \Delta S_{sysem}$$

$T = 100°C$

H_2O

600 kJ

$T_{surr} = 25°C$

Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

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Summary

- Entropy
- The Increase of entropy principle
- Some remarks about entropy
- Entropy change of pure substances
- Isentropic processes
- Property diagrams involving entropy
- What is entropy?
- The $T\,ds$ relations
- Entropy change of liquids and solids
- The entropy change of ideal gases
- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- Entropy balance