CLASS

Fourth Units (Second part)

Energy analysis of closed systems

MOVING BOUNDARY WORK

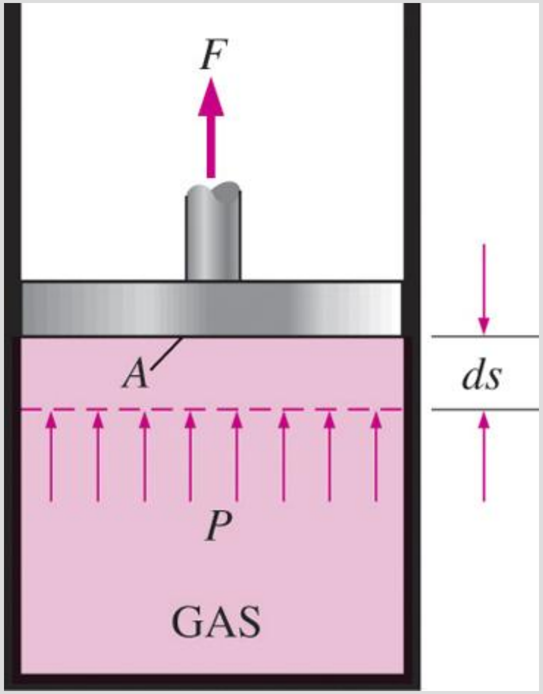
Moving boundary work (*P dV* work):
The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

$$W_b = \int_1^2 P\,dV \qquad (\text{kJ})$$

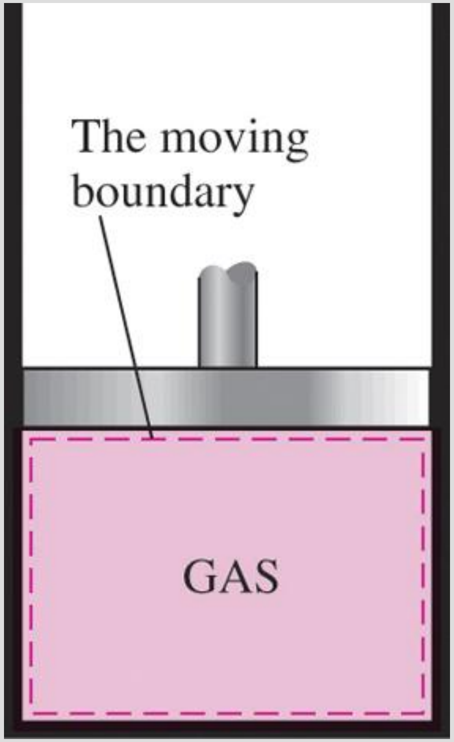
Quasi-equilibrium process:
A process during which the system remains nearly in equilibrium at all times.

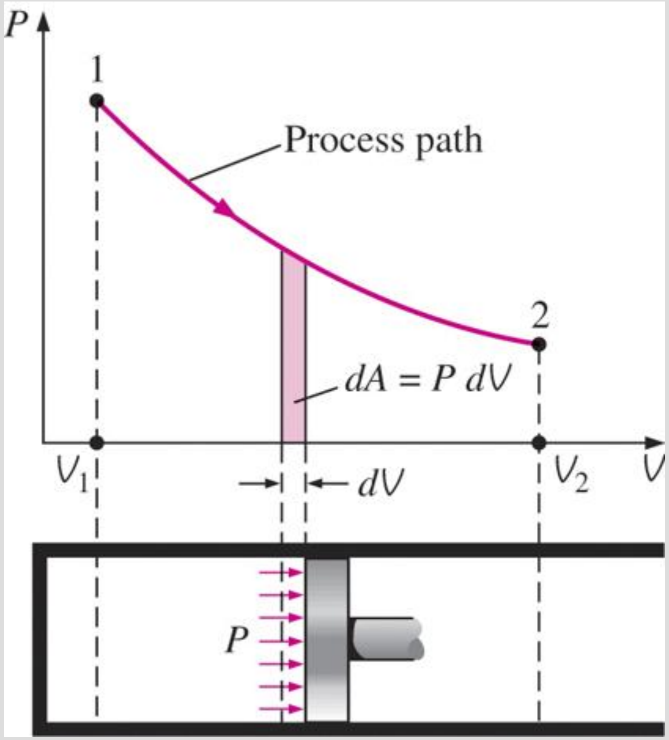
W_b is positive → for expansion
W_b is negative → for compression



A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount *ds*.

The work associated with a moving boundary is called *boundary work.*

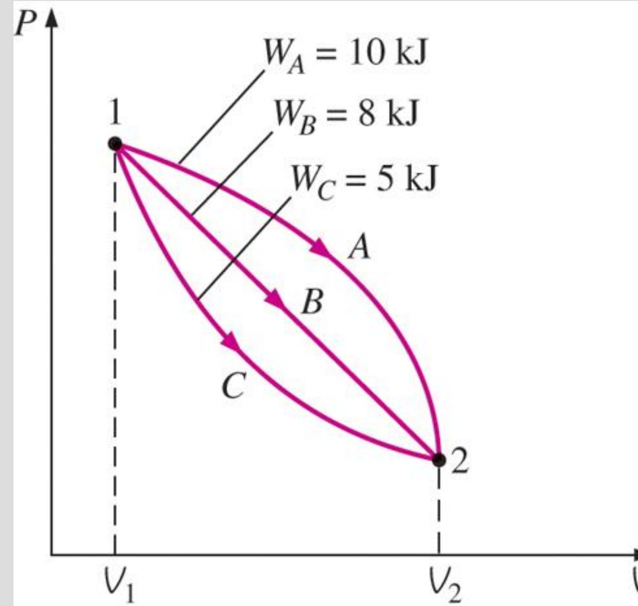




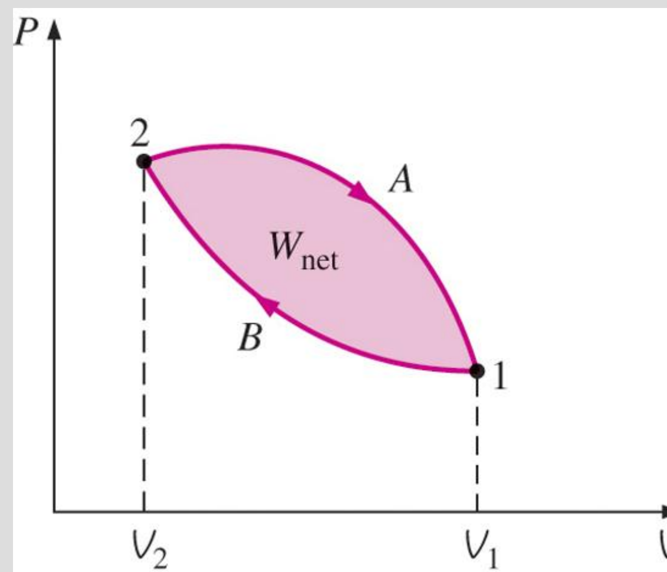
The area under the process curve on a *P-V* diagram represents the boundary work.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV$$

The boundary work done during a process depends on the path followed as well as the end states.

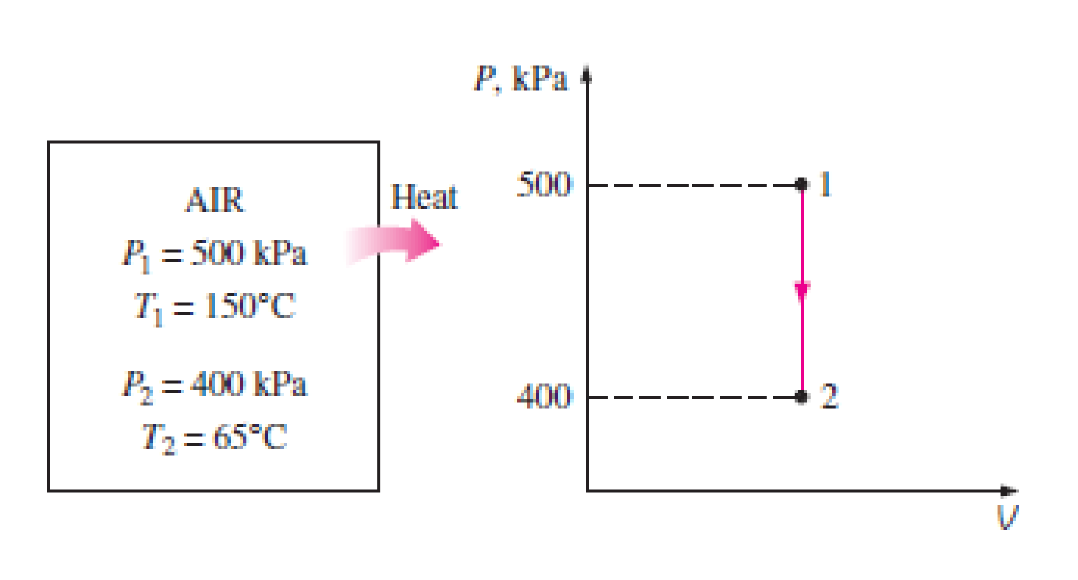


The net work done during a cycle is the difference between the work done by the system and the work done on the system.



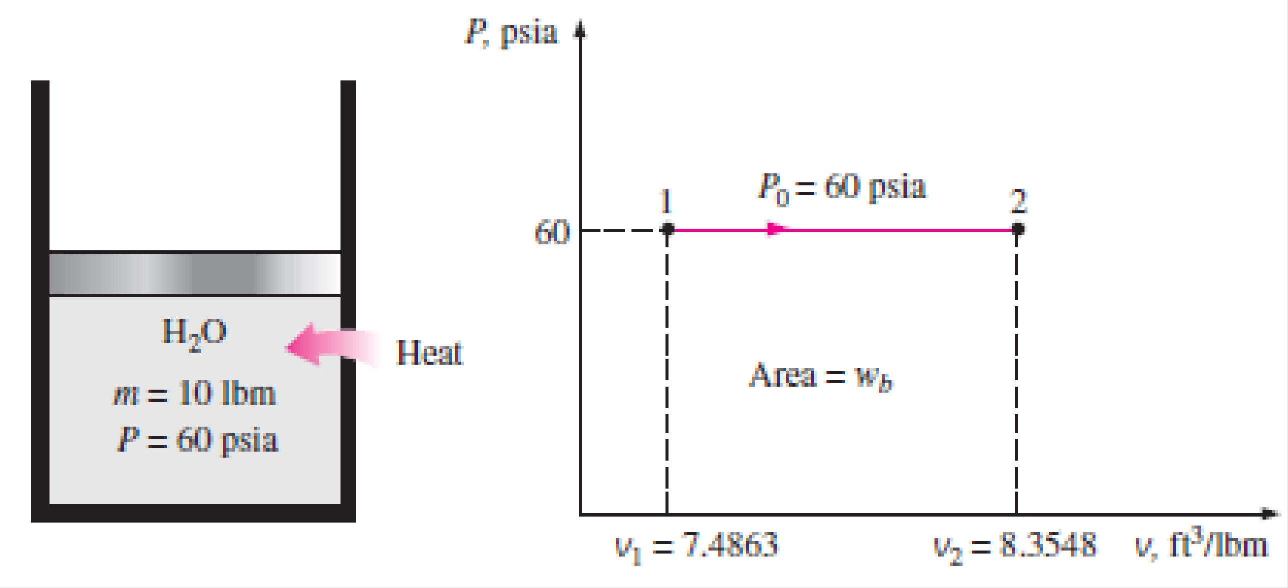
Boundary work for a constant-volume process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.



Boundary work for a constant-pressure process

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.



Polytropic, Isothermal, and Isobaric processes

$P = CV^{-n}$ Polytropic process: *C*, *n* (polytropic exponent) constants

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

Polytropic process

$$W_b = \frac{mR(T_2 - T_1)}{1-n}$$

Polytropic and for ideal gas

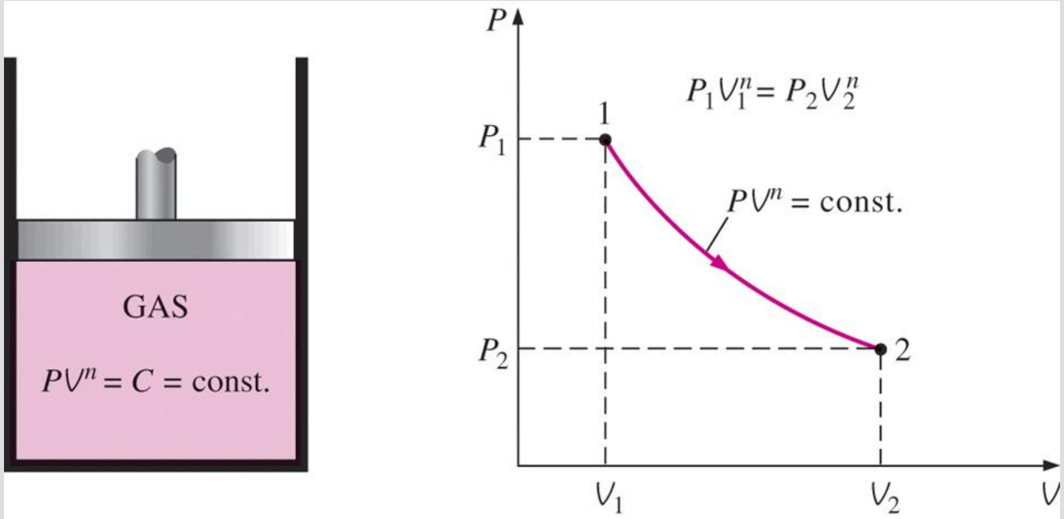
$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$

When $n = 1$ (isothermal process)

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$

Constant pressure process

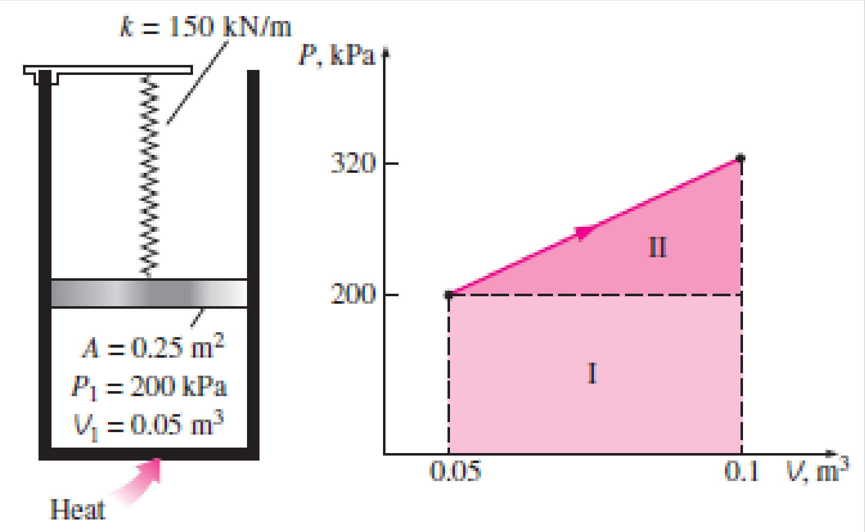
What is the boundary work for a constant-volume process?



Schematic and *P-V* diagram for a polytropic process.

Boundary work for a constant-pressure process

A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine (*a*) the final pressure inside the cylinder, (*b*) the total work done by the gas, and (*c*) the fraction of this work done against the spring to compress it.



ENERGY BALANCE FOR CLOSED SYSTEMS

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad \text{(kJ)}$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}} \quad \text{(kW)}$$

Energy balance in the rate form

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q}\,\Delta t, \quad W = \dot{W}\,\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t \quad \text{(kJ)}$$

$$e_{in} - e_{out} = \Delta e_{system} \quad \text{(kJ/kg)}$$

Energy balance per unit mass basis

$$\delta E_{in} - \delta E_{out} = dE_{system} \quad \text{or} \quad \delta e_{in} - \delta e_{out} = de_{system}$$

Energy balance in differential form

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in}$$

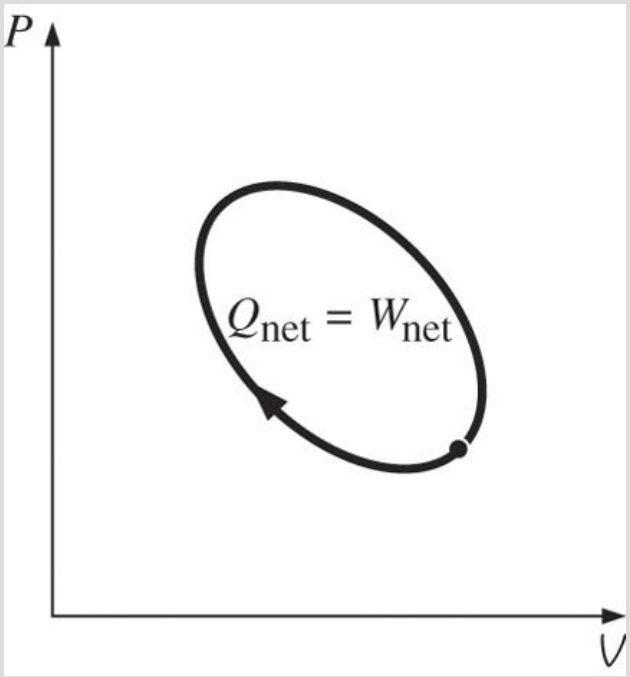
Energy balance for a cycle

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E$$

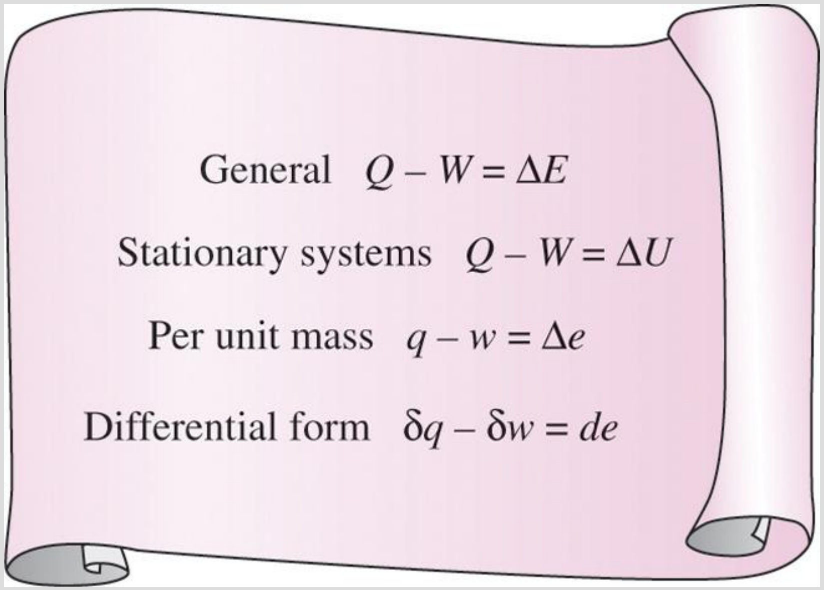
$$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$$

$$W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$$

Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).



For a cycle $\Delta E = 0$, thus $Q = W$.



Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \Delta KE^{\nearrow 0} + \Delta PE^{\nearrow 0}$$

$$Q - W_{other} - W_b = U_2 - U_1$$

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{other} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{other} = H_2 - H_1$$

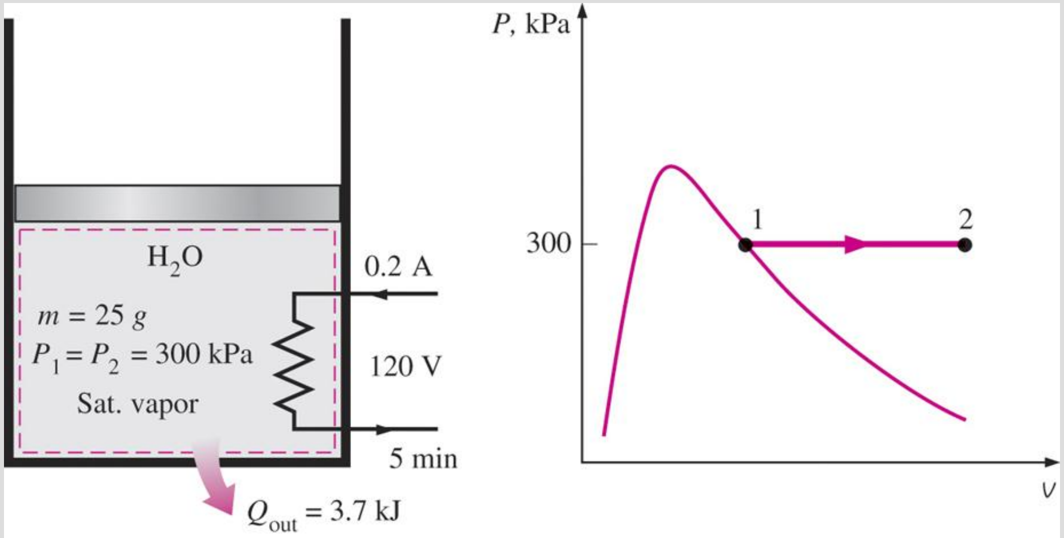
For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e,in} - Q_{out} - W_b = \Delta U$$

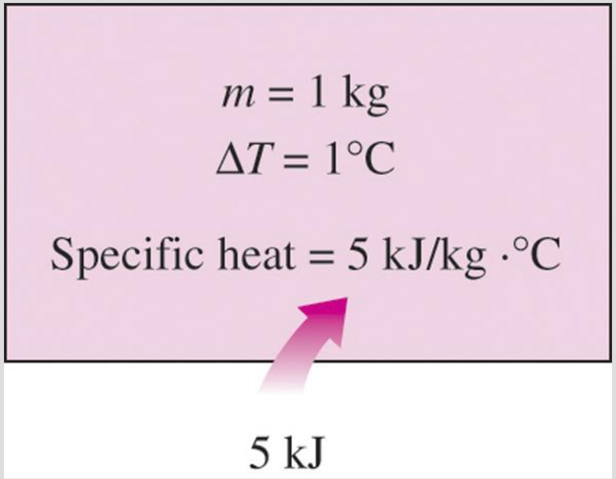
$$W_{e,in} - Q_{out} = \Delta H = m(h_2 - h_1)$$



SPECIFIC HEATS

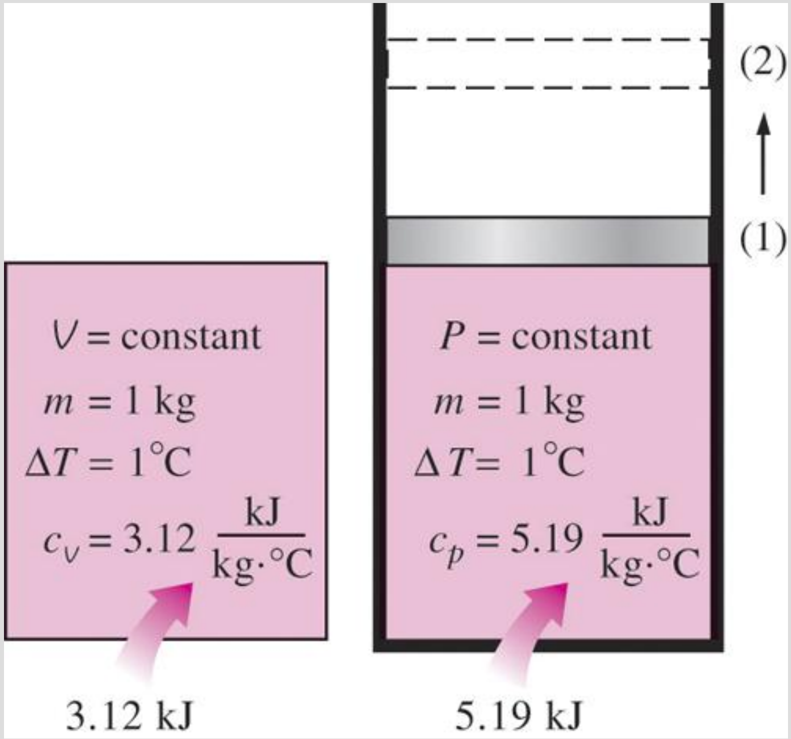
Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

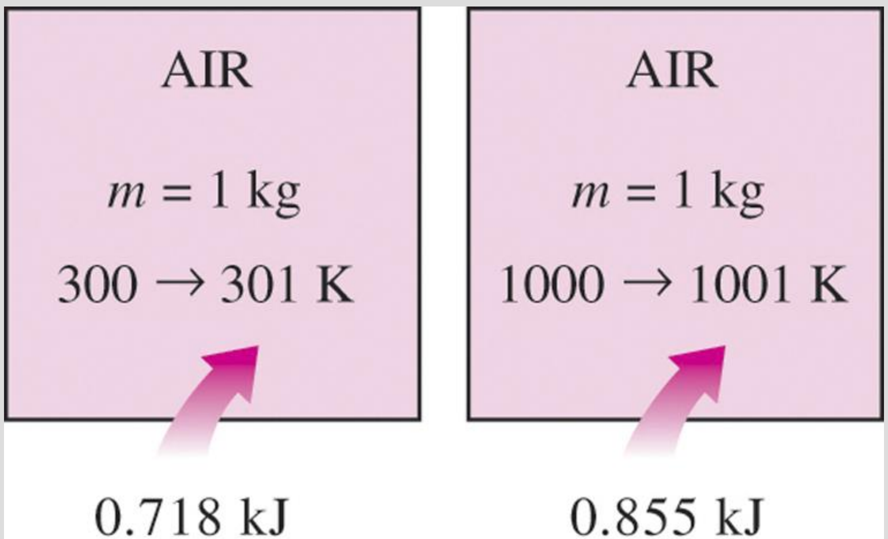


Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).



- The equations in the figure are valid for *any* substance undergoing *any* process.
- c_v and c_p are properties.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- A common unit for specific heats is kJ/kg · °C or kJ/kg · K. **Are these units identical?**



The specific heat of a substance changes with temperature.

True or False?
c_p is always greater than c_v.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

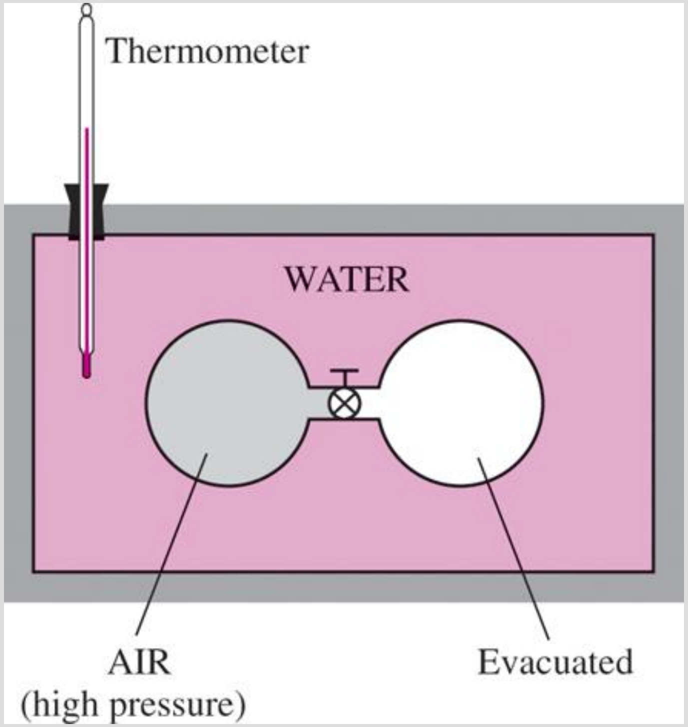
= the change in internal energy with temperature at constant volume

Formal definitions of c_v and c_p.

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES



$$\left.\begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned}\right\} \quad h = u + RT$$

$$u = u(T) \qquad h = h(T)$$

$$du = c_v(T)\, dT \qquad dh = c_p(T)\, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\, dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\, dT$$

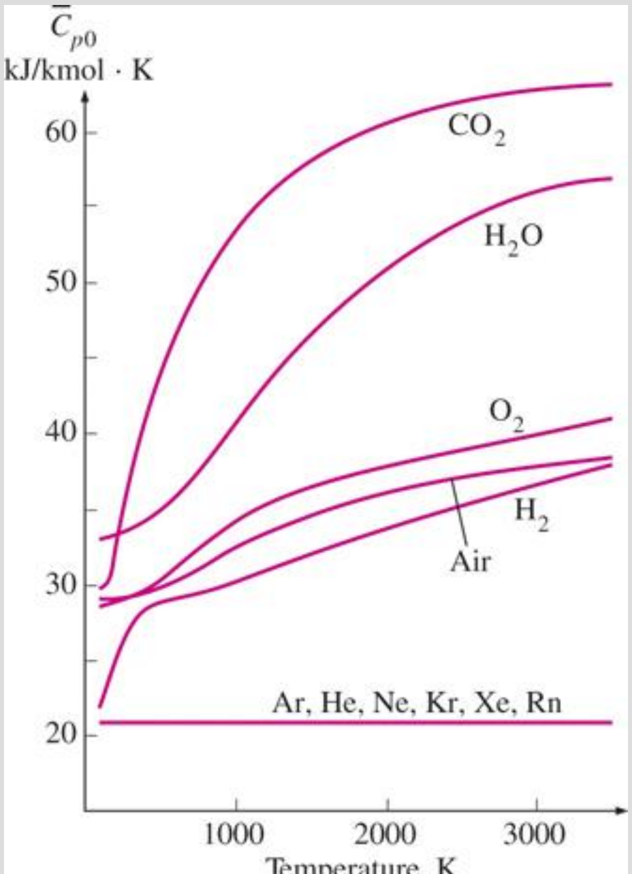
Joule showed using this experimental apparatus that $u=u(T)$

$$u = u(T)$$
$$h = h(T)$$
$$c_v = c_v(T)$$
$$c_p = c_p(T)$$

For ideal gases, u, h, c_v, and c_p vary with temperature only.

Internal energy and enthalpy change of an ideal gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.



Ideal-gas constant-pressure specific heats for some gases (see Table A–2*c* for c_p equations).

- *u* and *h* data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

AIR

T, K	u, kJ/kg	h, kJ/kg
0	0	0
·	·	·
·	·	·
300	214.07	300.19
310	221.25	310.24
·	·	·
·	·	·

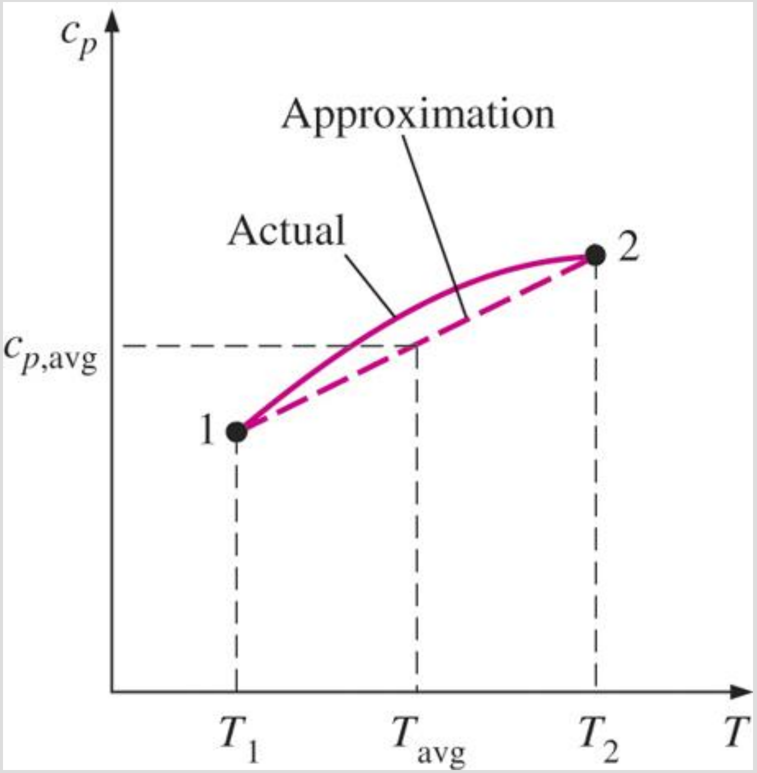
In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Internal energy and enthalpy change when specific heat is taken constant at an average value

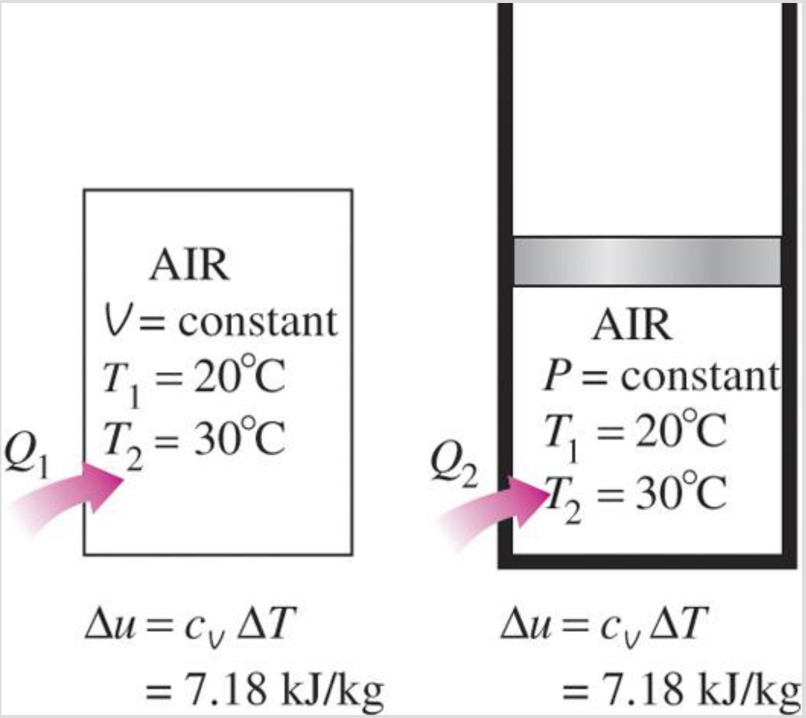
$$u_2 - u_1 = c_{v,\text{avg}}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,\text{avg}}(T_2 - T_1)$$

(kJ/kg)



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.



The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not.

Three ways of calculating Δu *and* Δh

1. By using the tabulated *u* and *h* data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \quad \text{(table)}$$

$$\Delta u = \int_1^2 c_v(T)\, dT$$

$$\Delta u \cong c_{v,\mathrm{avg}}\, \Delta T$$

Three ways of calculating Δu.

Specific Heat Relations of Ideal Gases

$$h = u + RT$$

$$dh = du + R\,dT$$

$dh = c_p dT$ and $du = c_v dT$

→ The relationship between c_p, c_v and R

$$c_p = c_v + R \qquad (\text{kJ/kg}\cdot\text{K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\text{kJ/kmol}\cdot\text{K})$$

AIR at 300 K

$$\left.\begin{aligned} c_v &= 0.718\ \text{kJ/kg}\cdot\text{K} \\ R &= 0.287\ \text{kJ/kg}\cdot\text{K} \end{aligned}\right\} c_p = 1.005\ \text{kJ/kg}\cdot\text{K}$$

or

$$\left.\begin{aligned} \bar{c}_v &= 20.80\ \text{kJ/kmol}\cdot\text{K} \\ R_u &= 8.314\ \text{kJ/kmol}\cdot\text{K} \end{aligned}\right\} \bar{c}_p = 29.114\ \text{kJ/kmol}\cdot\text{K}$$

The c_p of an ideal gas can be determined from a knowledge of c_v and R.

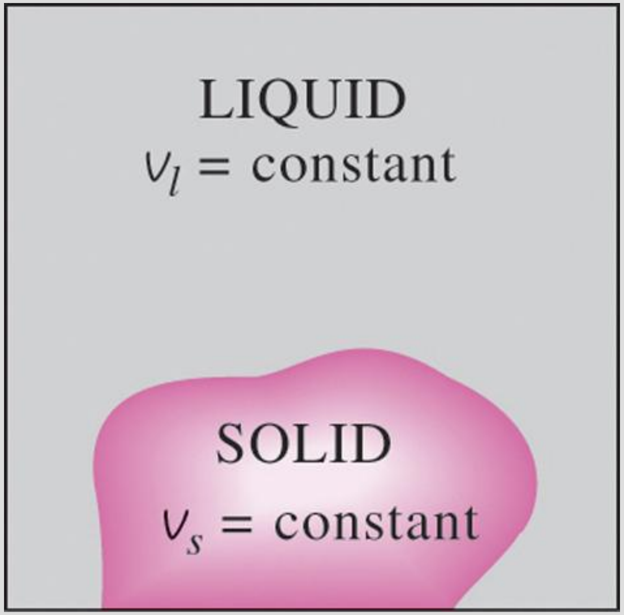
$$k = \frac{c_p}{c_v}$$

Specific heat ratio

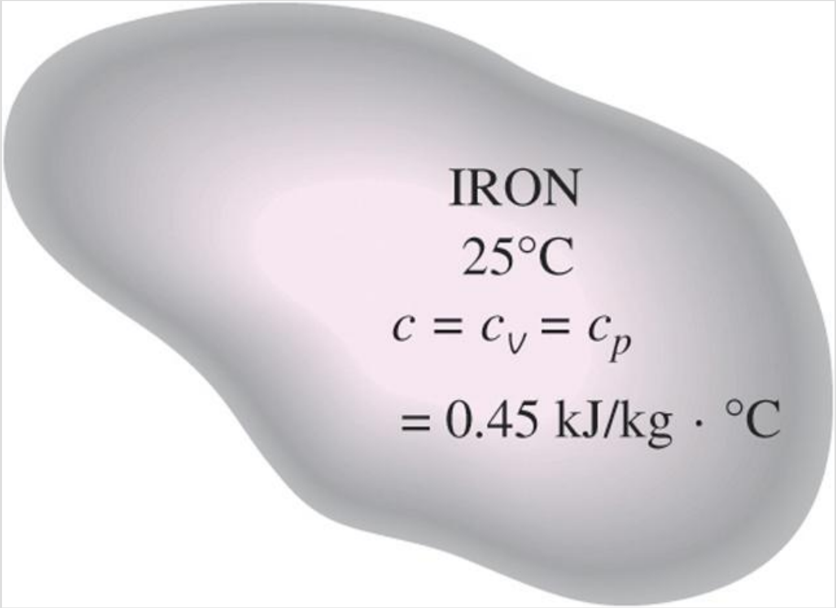
- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Incompressible substance: A substance whose specific volume (or density) is constant. Solids and liquids are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by *c*.

Internal Energy Changes

$$du = c_v\, dT = c(T)\, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad \text{(kJ/kg)}$$

$$\Delta u \cong c_{avg}(T_2 - T_1) \qquad \text{(kJ/kg)}$$

Enthalpy Changes

$$h = u + Pv$$

$$dh = du + v\, dP + P\, dv^{\nearrow 0} = du + v\, dP$$

$$\Delta h = \Delta u + v\, \Delta P \cong c_{avg}\, \Delta T + v\, \Delta P \qquad \text{(kJ/kg)}$$

For *solids*, the term $v\, \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{avg}\Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{avg}\Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v\, \Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{sat\,@\,T})$$

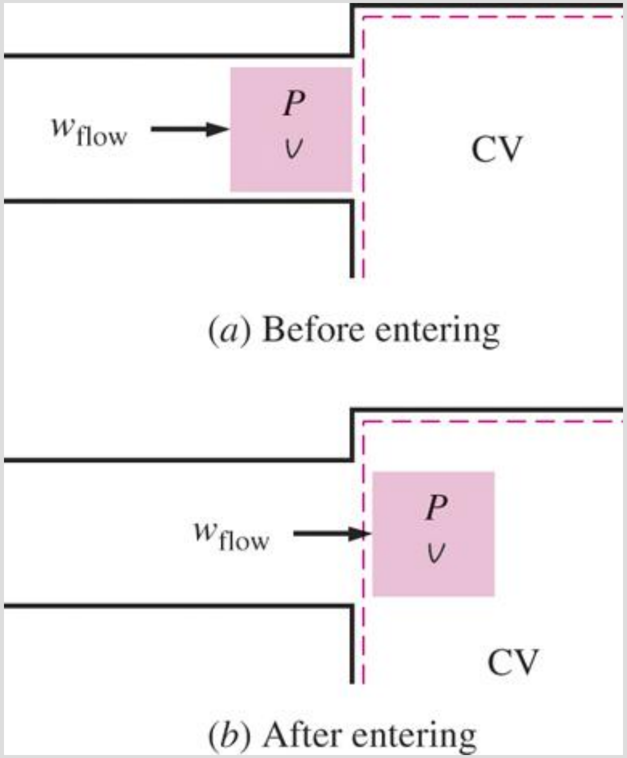
The enthalpy of a compressed liquid

A more accurate relation than $h_{@P,T} \cong h_{f@T}$

Energy analysis of open systems

FLOW WORK AND THE ENERGY OF A FLOWING FLUID

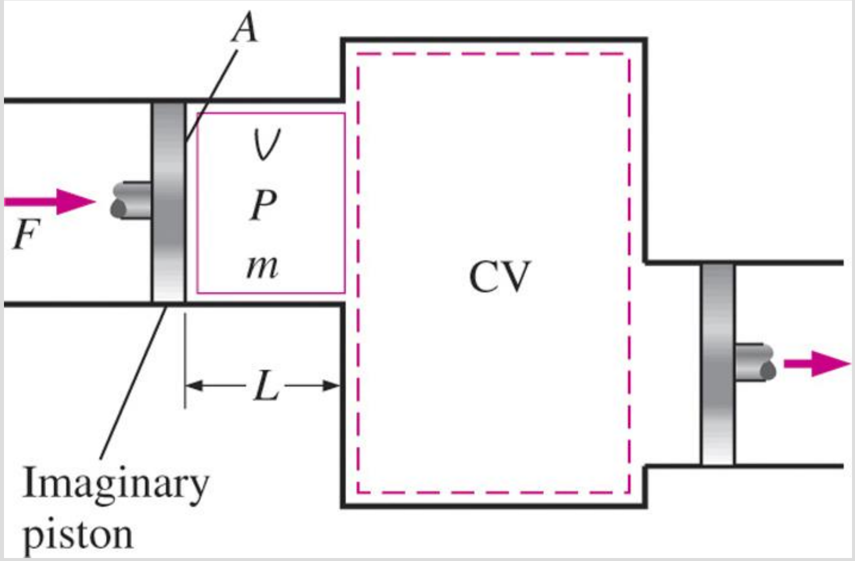
Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.



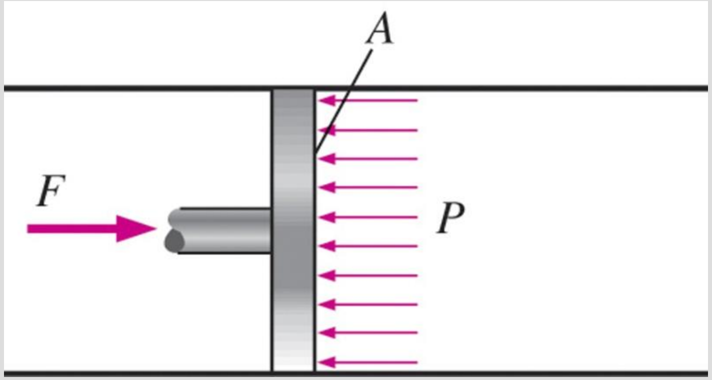
$$F = PA$$

$$W_{flow} = FL = PAL = PV \qquad (kJ)$$

$$w_{flow} = PV \qquad (kJ/kg)$$



Schematic for flow work.



In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

Total Energy of a Flowing Fluid

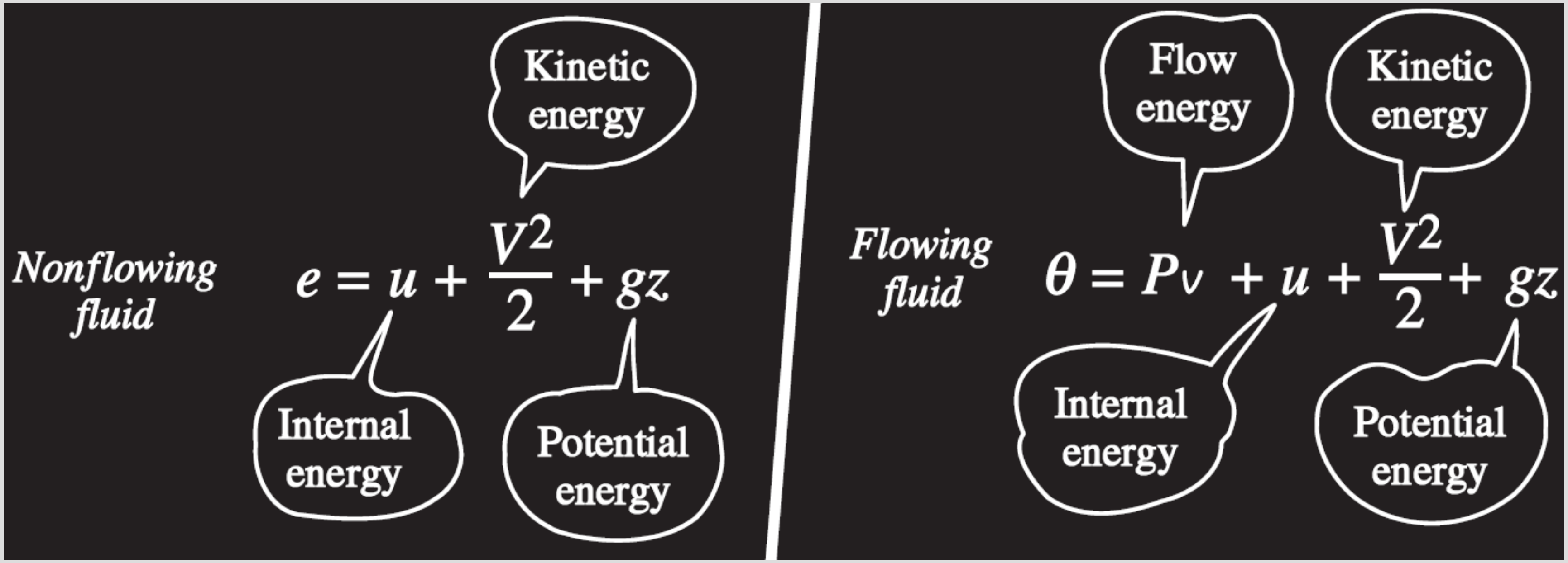
$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)}$$

$$\theta = Pv + e = Pv + (u + \text{ke} + \text{pe})$$

$$h = u + Pv$$

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)}$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.

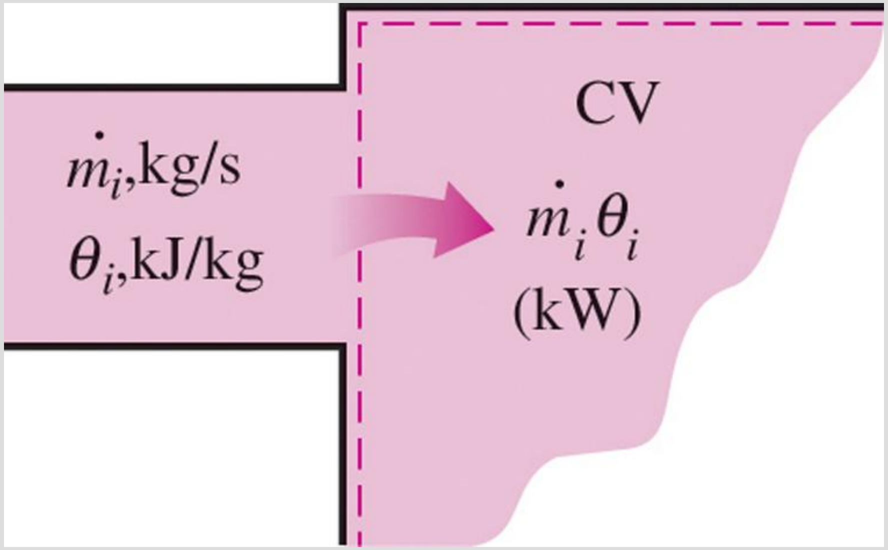


The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

Energy Transport by Mass

Amount of energy transport: $$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kJ)}$$

Rate of energy transport: $$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kW)}$$



The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh \qquad \dot{E}_{\text{mass}} = \dot{m}h$$

When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\text{in,mass}} = \int_{m_i} \theta_i \,\delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

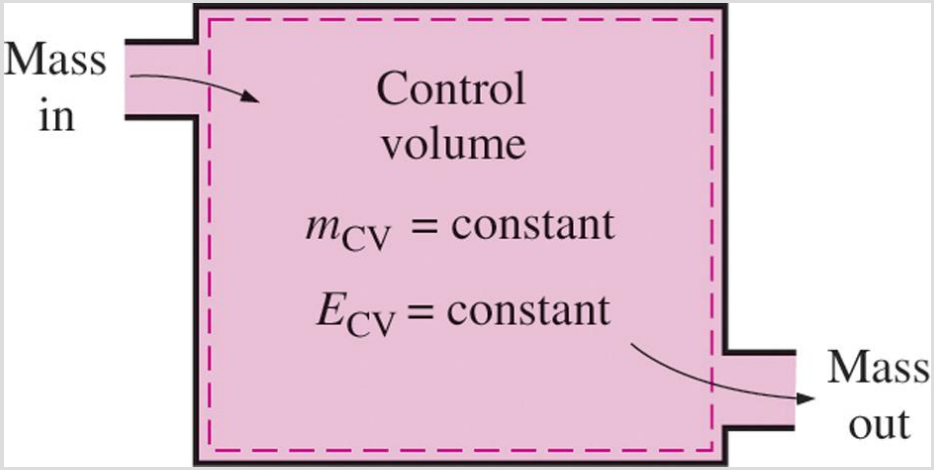
ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz \right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz \right)$$

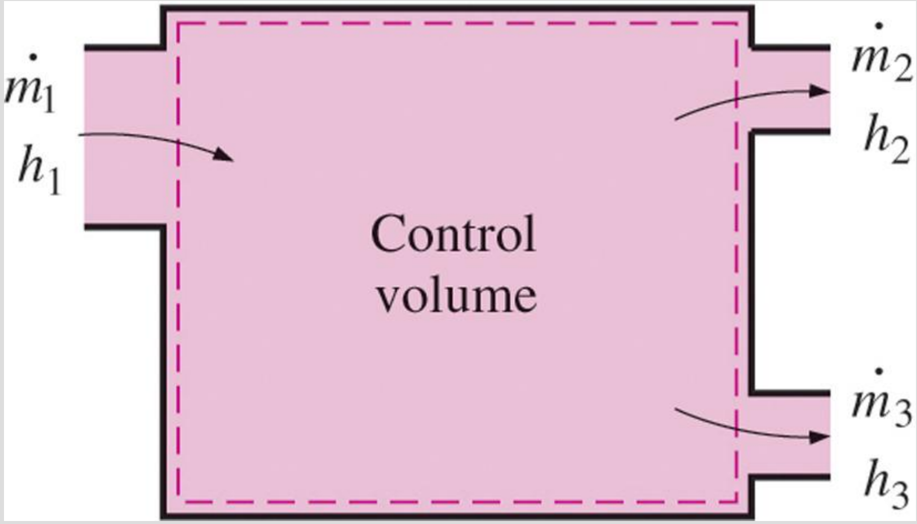
- For steady flow, time rate of change of the energy content of the CV is zero.
- This equation states: *the net rate of energy transfer to a CV by heat and work transfers during steady flow is equal to the difference between the rates of outgoing* and incoming energy flows with mass.

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

Many engineering systems such as power plants operate under steady conditions.



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

Mass and Energy balances for a steady-flow process

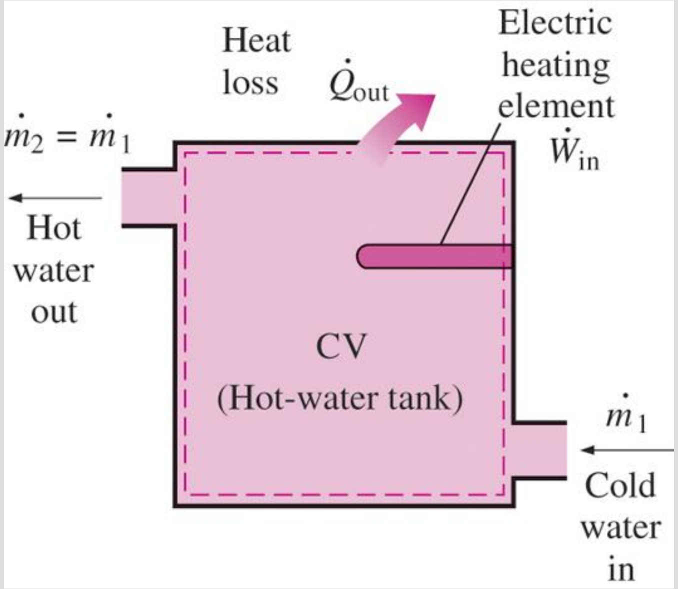
$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad (kg/s)$$

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Mass balance

A water heater in steady operation.



Energy balance

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{system}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{\nearrow\ 0\ (\text{steady})}{} = 0$$

$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{out}}_{\text{Rate of net energy transfer out by heat, work, and mass}} \qquad (kW)$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}}$$

Energy balance relations with sign conventions (i.e., heat input and work output are positive)

$$\dot{Q} - \dot{W} = \underbrace{\sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \underbrace{\sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}}$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

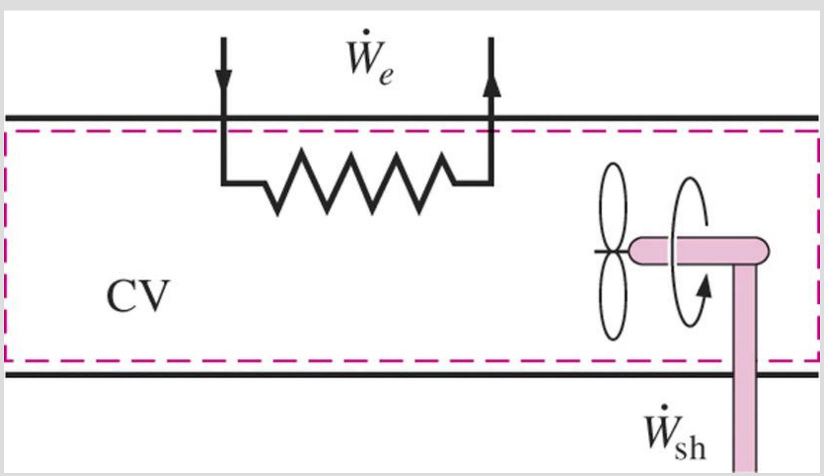
$$q - w = h_2 - h_1 \qquad q = \dot{Q}/\dot{m} \qquad w = \dot{W}/\dot{m}$$

when kinetic and potential energy changes are negligible

$$\frac{\text{J}}{\text{kg}} \equiv \frac{\text{N}\cdot\text{m}}{\text{kg}} \equiv \left(\text{kg}\frac{\text{m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2}$$

$$\left(\text{Also, } \frac{\text{Btu}}{\text{lbm}} \equiv 25{,}037\ \frac{\text{ft}^2}{\text{s}^2}\right)$$

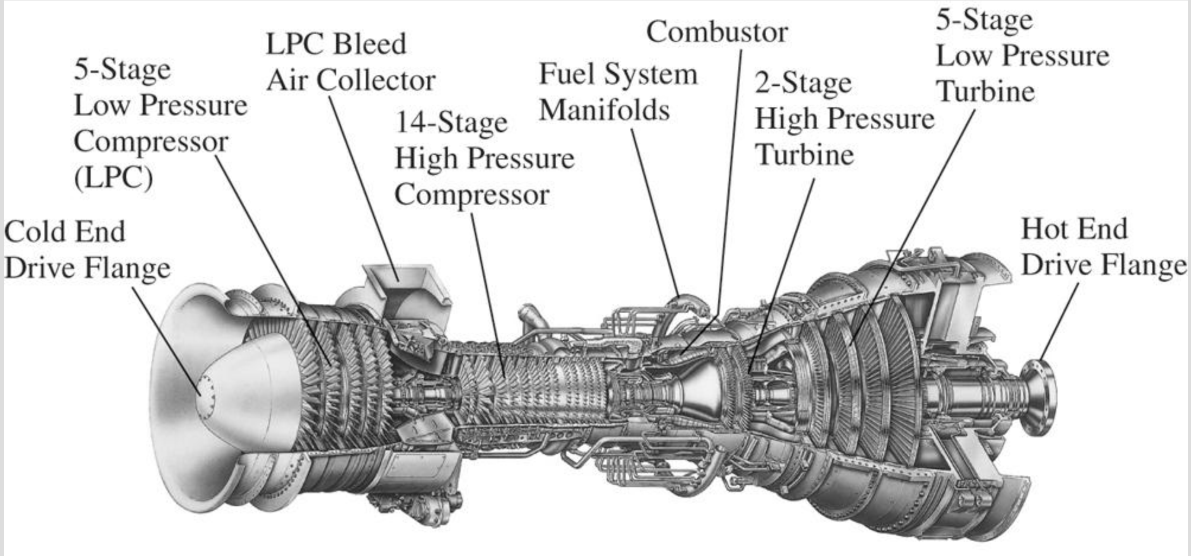
Some energy unit equivalents



Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

SOME STEADY-FLOW ENGINEERING DEVICES

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance. Therefore, these devices can be conveniently analyzed as steady-flow devices.

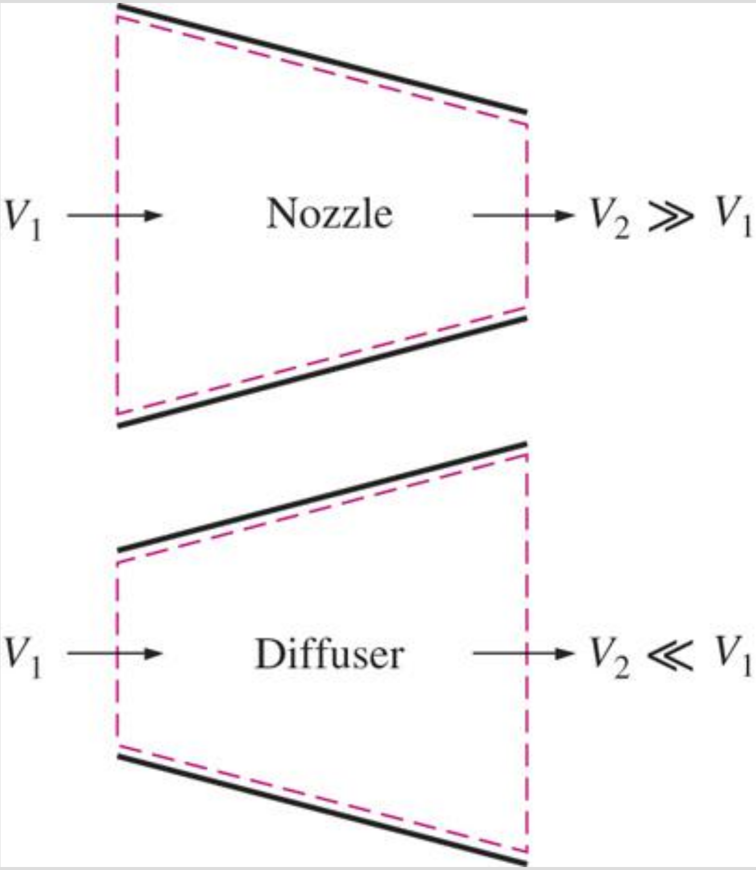


A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

V_1 m/s	V_2 m/s	Δke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.

Nozzles and Diffusers



Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

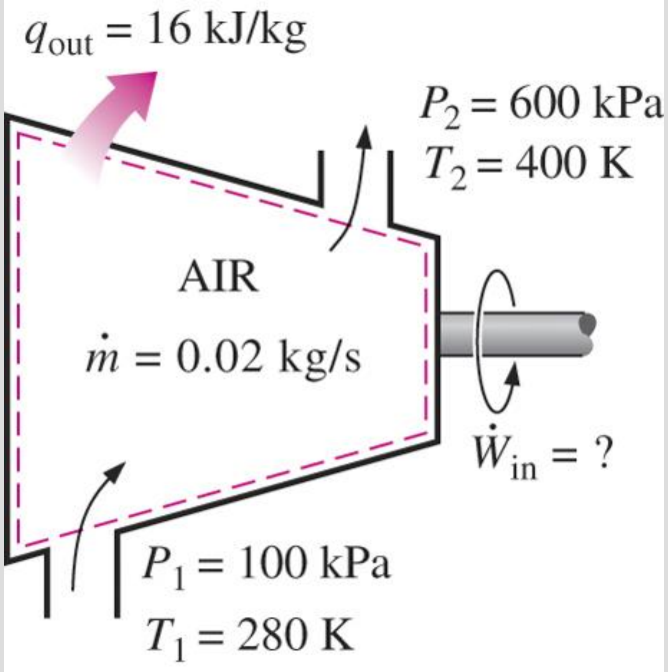
Energy balance for a nozzle or diffuser:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

Turbines and Compressors



Energy balance for the compressor in this figure:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$(\text{since } \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

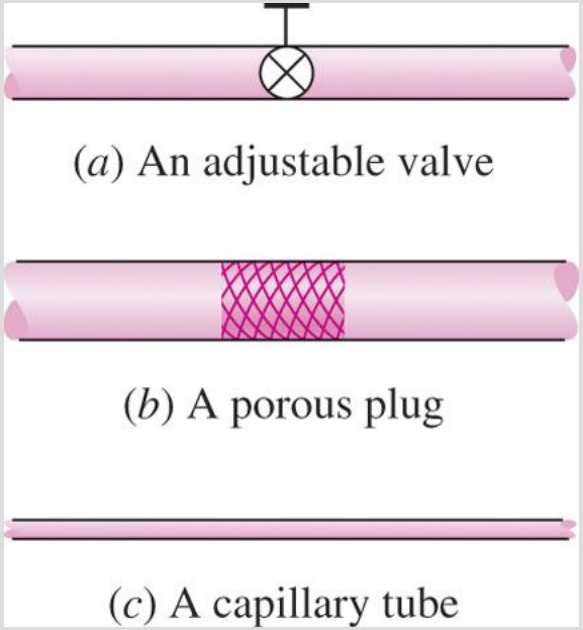
- **Turbine** drives the electric generator In steam, gas, or hydroelectric power plants.
- As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.
- **Compressors**, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.
- A ***fan*** increases the pressure of a gas slightly and is mainly used to mobilize a gas.
- A ***compressor*** is capable of compressing the gas to very high pressures.
- ***Pumps*** work very much like compressors except that they handle liquids instead of gases.

Throttling valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.


(*a*) An adjustable valve

(*b*) A porous plug

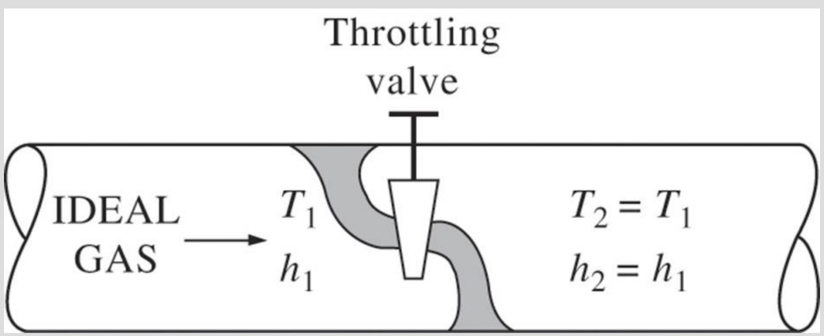
(*c*) A capillary tube

Energy balance

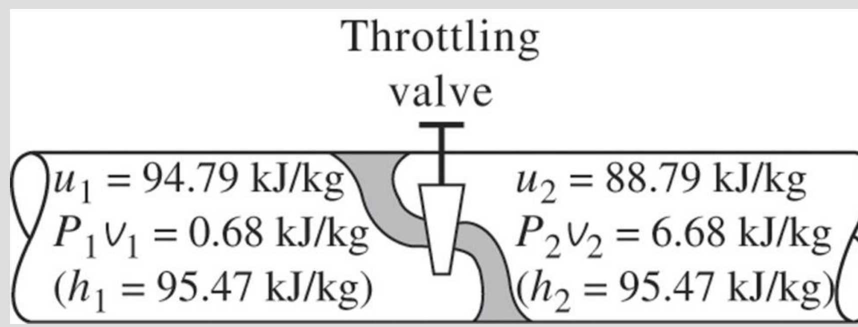
$$h_2 \cong h_1$$

$$u_1 + P_1v_1 = u_2 + P_2v_2$$

$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$



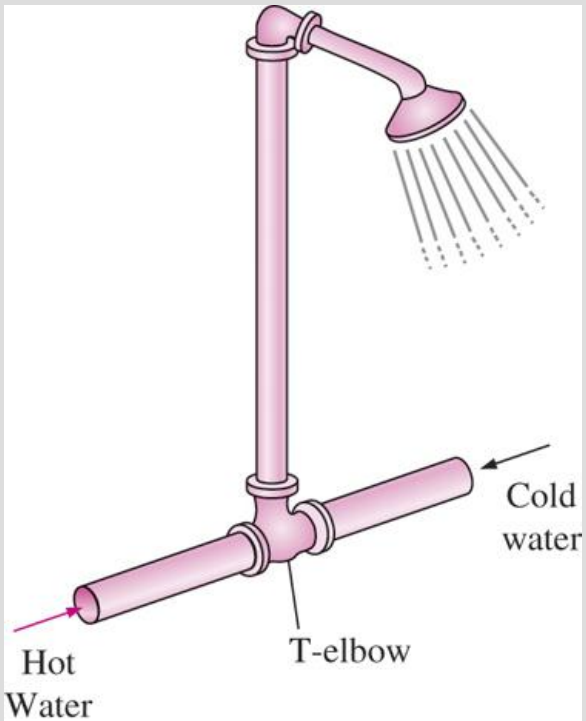
The temperature of an ideal gas does not change during a throttling (h = constant) process since $h = h(T)$.



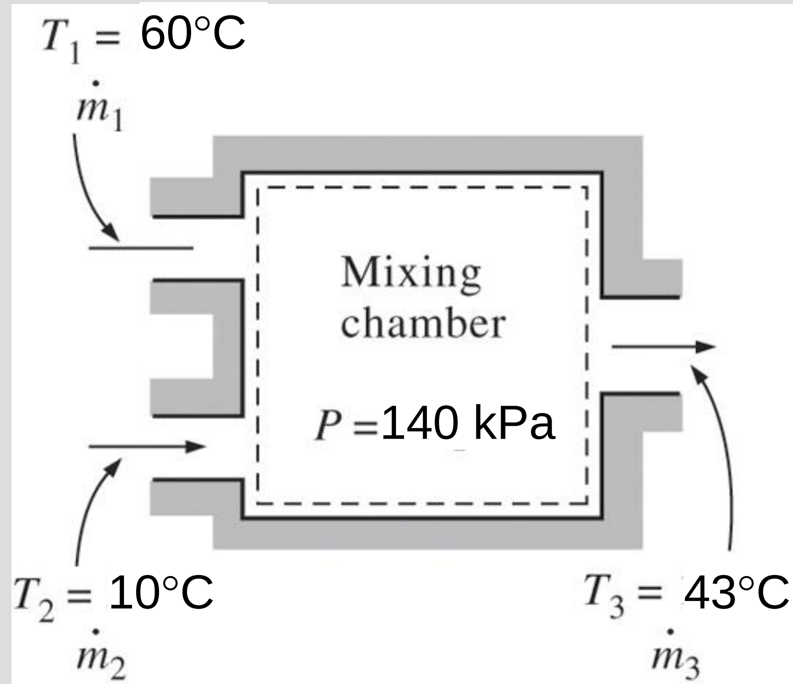
During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

Mixing chambers

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.



The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



Energy balance for the adiabatic mixing chamber in the figure is:

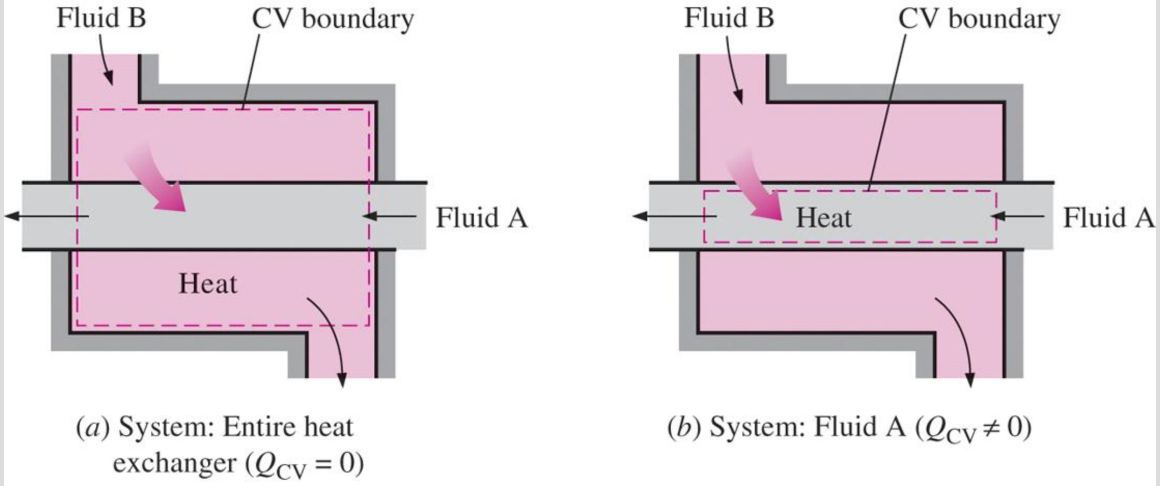
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

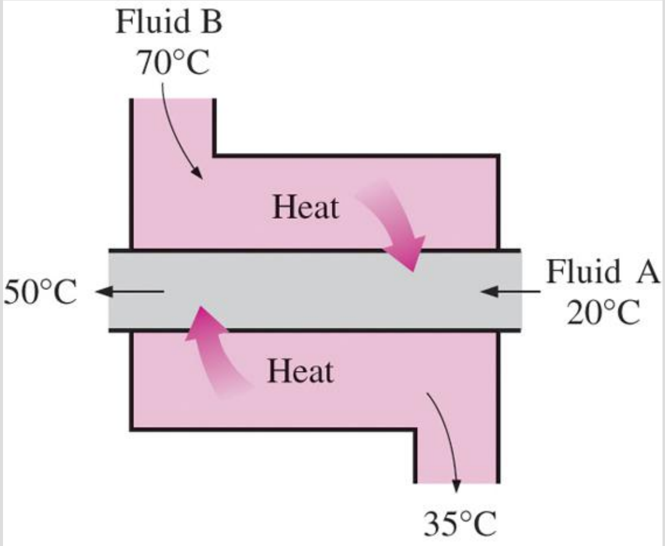
(since $\dot{Q} \cong 0$, $\dot{W} = 0$, ke $\cong$ pe $\cong 0$)

Heat exchangers

Heat exchangers are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.



The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.



A heat exchanger can be as simple as two concentric pipes.

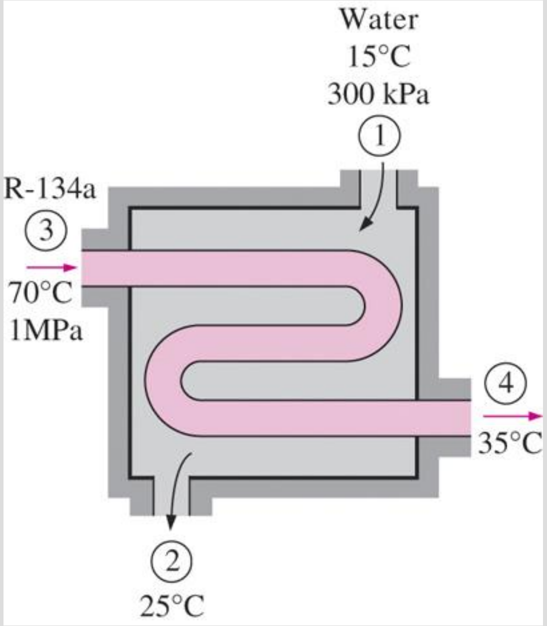
Mass and energy balances for the adiabatic heat exchanger in the figure is:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

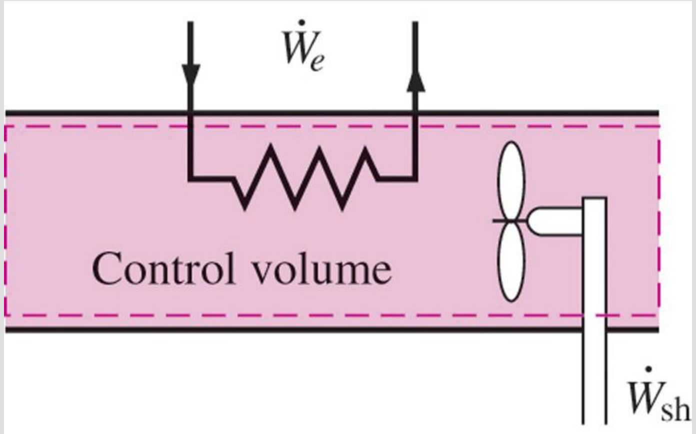
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

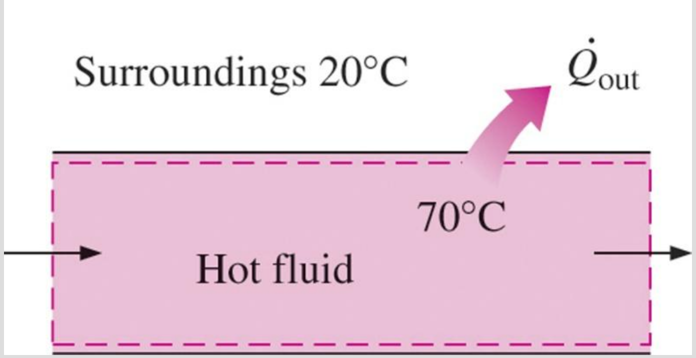


Pipe and duct fow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



Pipe or duct flow may involve more than one form of work at the same time.



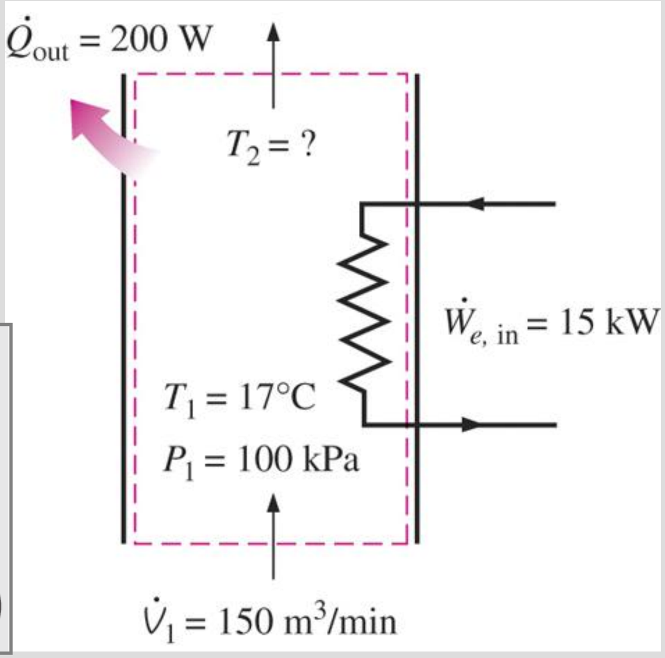
Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

Energy balance for the pipe flow shown in the figure is

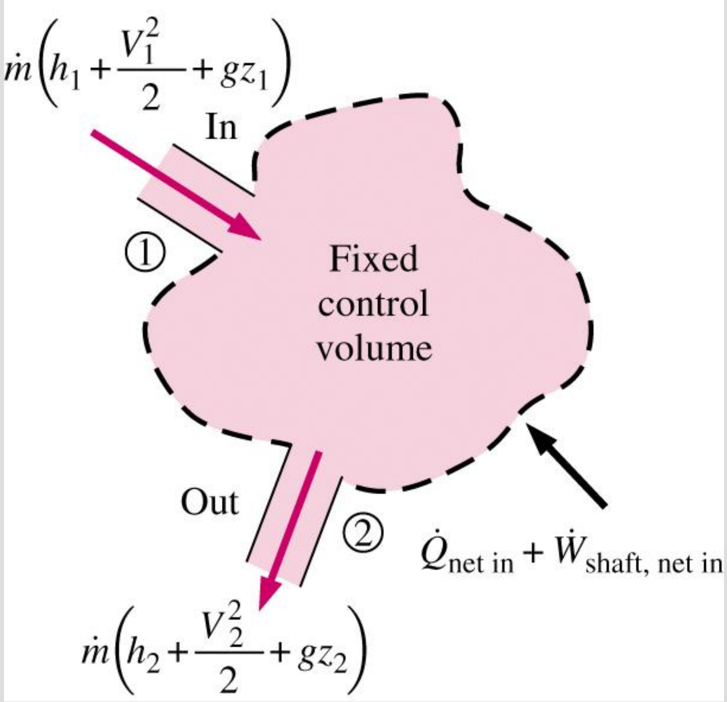
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$\dot{W}_{e,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$



ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS



- For **single-stream devices**, mass flow rate is constant.

$$q_{net,in}+w_{shaft,net,in}=h_2-h_1+\frac{V_2^2-V_1^2}{2}+g\left(z_2-z_1\right)$$

$$w_{shaft,net,in}+\frac{P_1}{\rho_1}+\frac{V_1^2}{2}+gz_1=\frac{P_2}{\rho_2}+\frac{V_2^2}{2}+gz_2+\left(u_2-u_1-q_{net,in}\right)$$

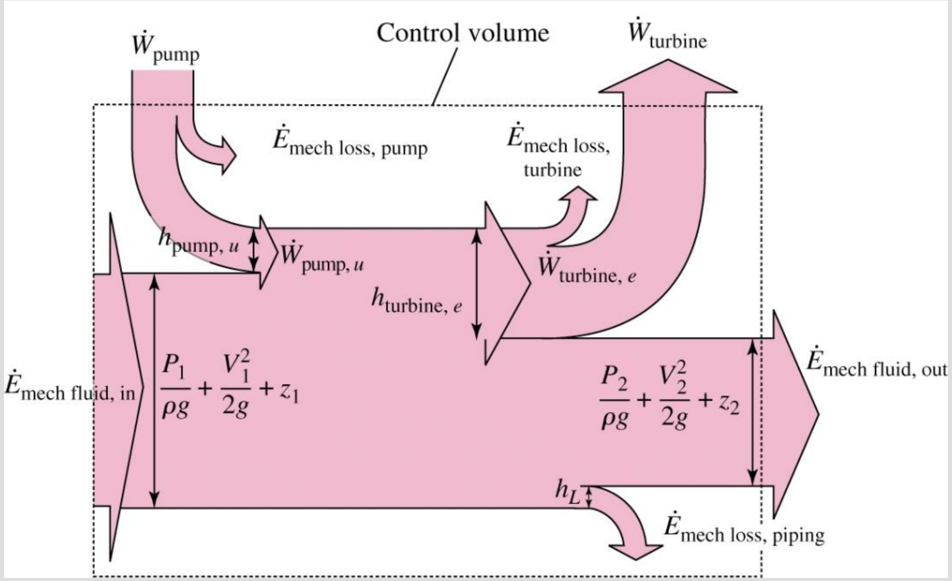
$$\frac{P_1}{\rho_1}+\frac{V_1^2}{2}+gz_1+w_{pump}=\frac{P_2}{\rho_2}+\frac{V_2^2}{2}+gz_2+w_{turbine}+e_{mech,loss}$$

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

- Divide by *g* to get each term in units of length

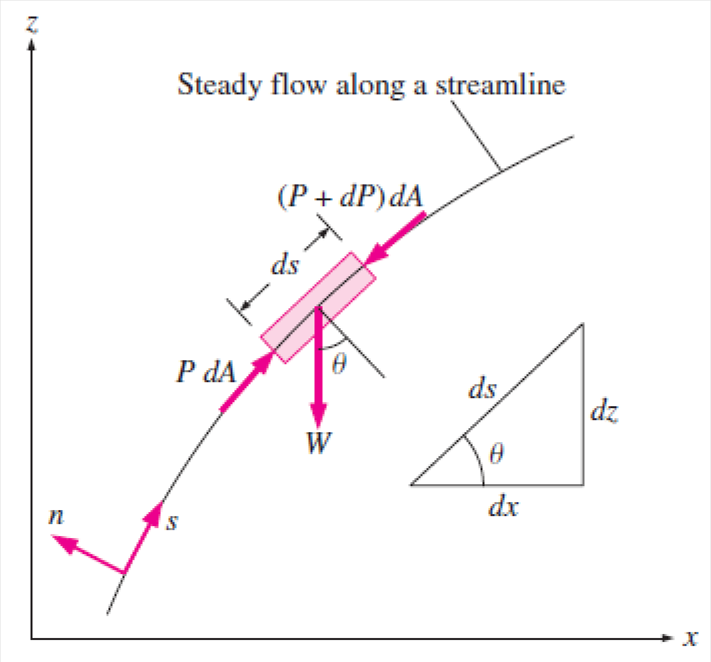
$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

- Magnitude of each term is now expressed as an equivalent column height of fluid, i.e., *Head*



The Bernoulli Equation

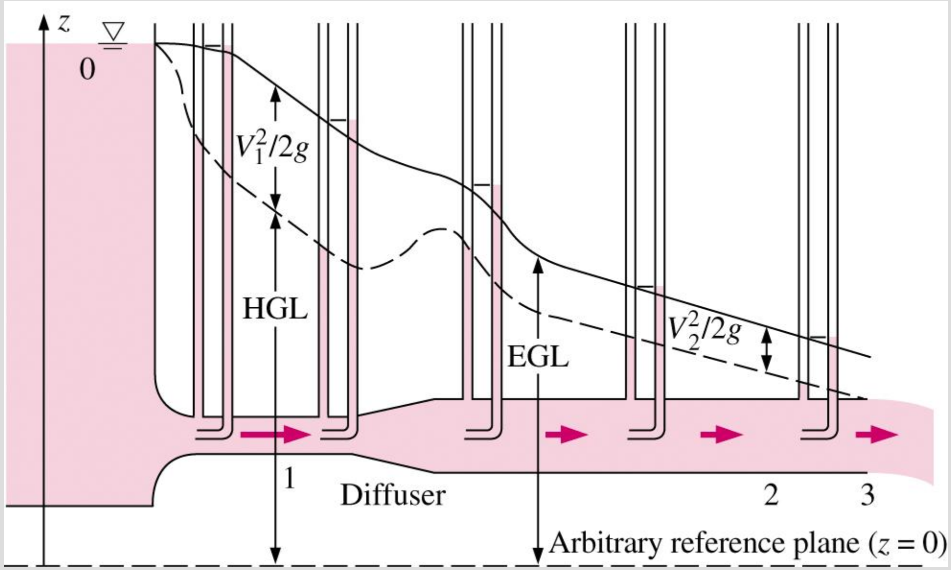
- If we neglect piping losses, and have a system without pumps or turbines



$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2$$

- This is the **Bernoulli equation**
- It can also be derived using Newton's second law of motion (see text, p. 187).
- 3 terms correspond to: Static, dynamic, and hydrostatic head (or pressure).

HGL and EGL



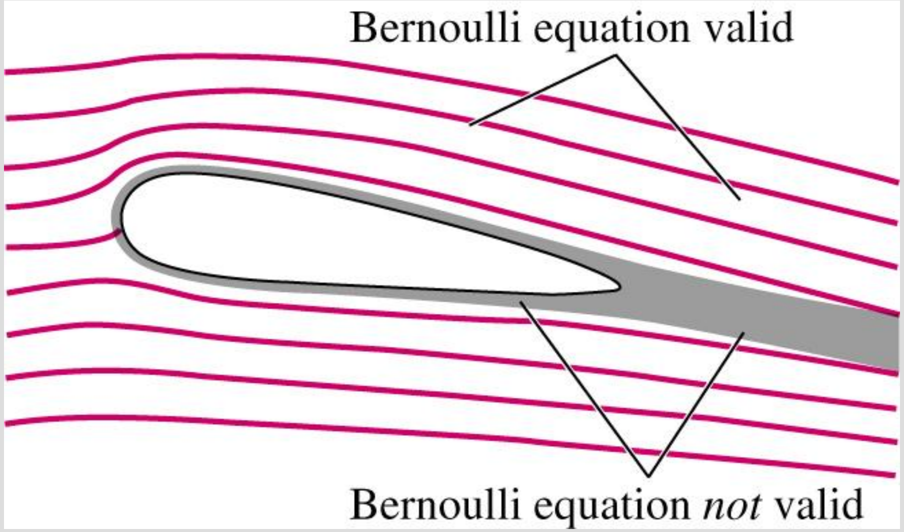
- It is often convenient to plot mechanical energy graphically using heights.
- Hydraulic Grade Line

$$HGL = \frac{P}{\rho g} + z$$

- Energy Grade Line (or total energy)

$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

The Bernoulli Equation



- The **Bernoulli equation** is an *approximate relation between pressure, velocity*, and *elevation and is valid in regions of steady, incompressible flow where net frictional forces are negligible.*
- Equation is useful in flow regions outside of boundary layers and wakes.

The Bernoulli Equation

- Limitations on the use of the Bernoulli Equation
  - ✓ Steady flow: $d/dt = 0$
  - ✓ Frictionless flow
  - ✓ No shaft work: $w_{pump} = w_{turbine} = 0$
  - ✓ Incompressible flow: ρ = constant
  - ✓ No heat transfer: $q_{net,in} = 0$
  - ✓ Applied along a streamline (except for irrotational flow)

Mechanical Energy

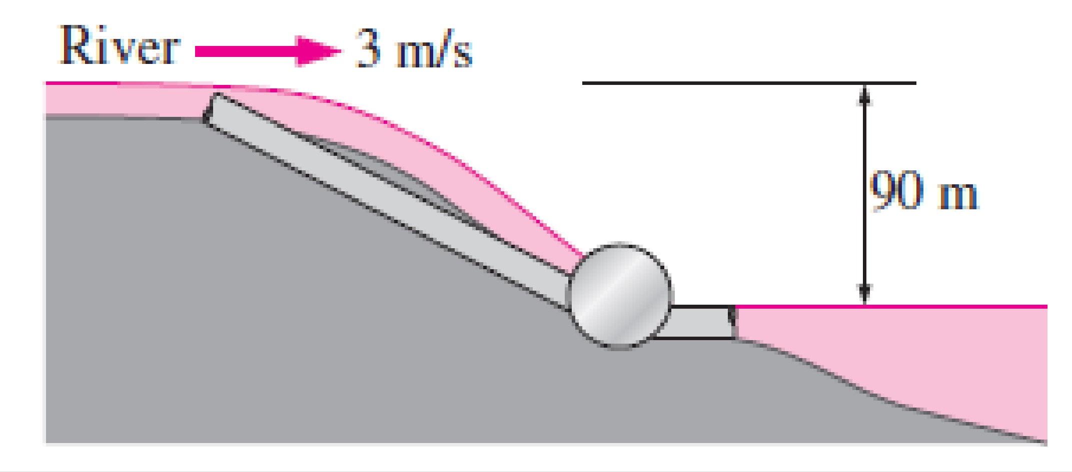
- **Mechanical energy** can be defined as *the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.*
- Flow P/ρ, kinetic V^2/g, and potential gz energy are the forms of mechanical energy $e_{mech} = P/\rho + V^2/g + gz$
- Mechanical energy change of a fluid during incompressible flow becomes

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

- In the absence of loses, Δe_{mech} represents the work supplied to the fluid ($\Delta e_{mech} > 0$) or extracted from the fluid ($\Delta e_{mech} < 0$).

example

Consider a river flowing toward a lake at an average velocity of 3 m/s at a rate of 500 m^3/s at a location 90 m above the lake surface. Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location.



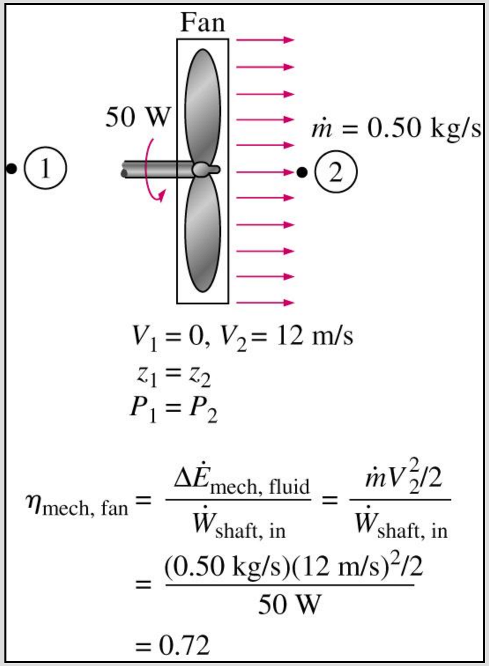
Efficiency

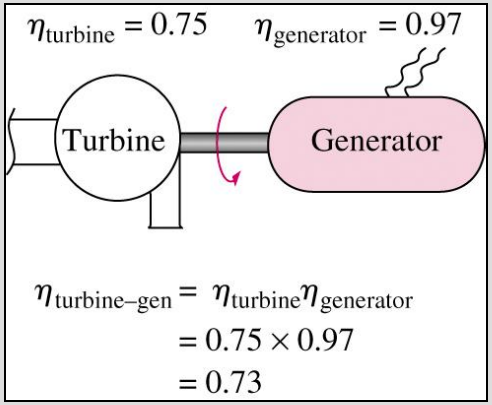
- Transfer of e_{mech} is usually accomplished by a rotating shaft: *shaft work*
- Pump, fan, propulsion: receives shaft work (e.g., from an electric motor) and transfers it to the fluid as mechanical energy
- Turbine: converts e_{mech} of a fluid to shaft work.
- In the absence of irreversibilities (e.g., friction), **mechanical efficiency** of a device or process can be defined as

$$\eta_{mech} = \frac{E_{mech,out}}{E_{mech,in}} = 1 - \frac{E_{mech,loss}}{E_{mech,in}}$$

- If $\eta_{mech} < 100\%$, losses have occurred during conversion.

Pump and Turbine Efficiencies





- In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid.
- In these cases, efficiency is better defined as the ratio of (supplied or extracted work) vs. rate of increase in mechanical energy

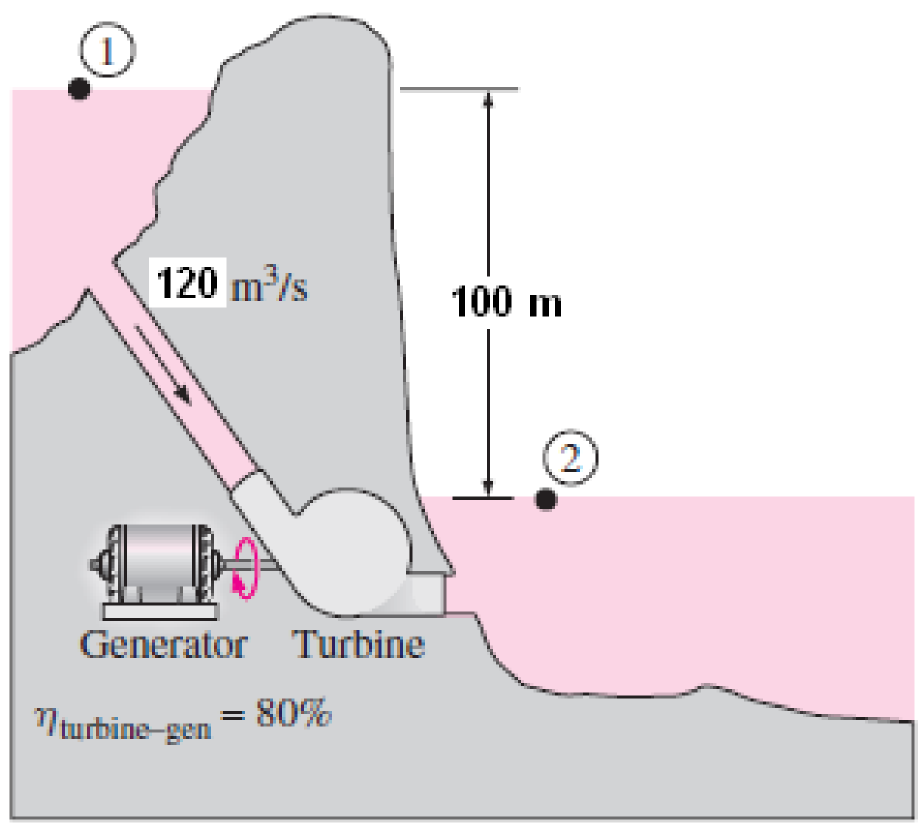
$$\eta_{pump} = \frac{\Delta \dot{E}_{mech,\ fluid}}{\dot{W}_{shaft,in}}$$

$$\eta_{turbine} = \frac{\dot{W}_{shaft,out}}{\left|\Delta \dot{E}_{mech,\ fluid}\right|}$$

- Overall efficiency must include motor or generator efficiency.

Example:

In a hydroelectric power plant, 120 m^3/s of water flows from an elevation of 100 m to a turbine, where electric power is generated. The overall efficiency of the turbine–generator is 80 percent. Disregarding frictional losses in piping, estimate the electric power output of this plant.



Example:

A garden hose attached with a nozzle is used to fill a 20-gal bucket. The inner diameter of the hose is 1 in and it reduces to 0.5 in at the nozzle exit. If the average velocity in the hose is 8 ft/s, determine:

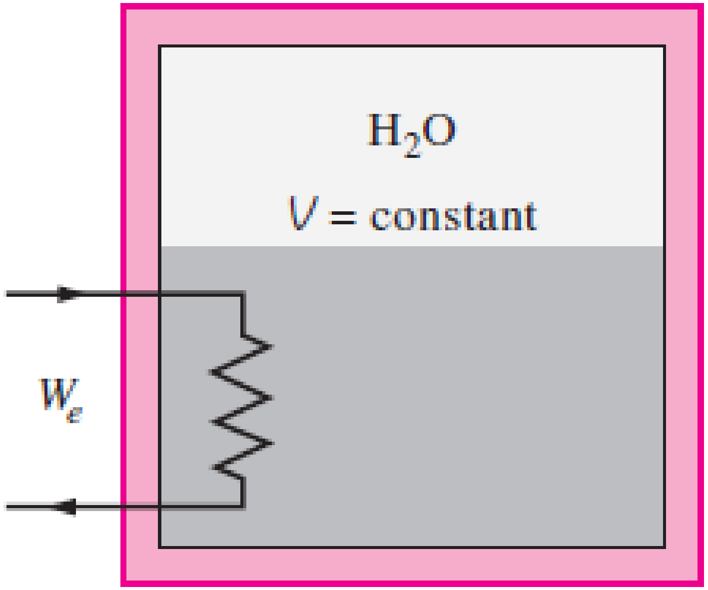
(a) the volume and mass flow rates of water through the hose

(b) how long it will take to fill the bucket with water, and

(c) the average velocity of water at the nozzle exit.

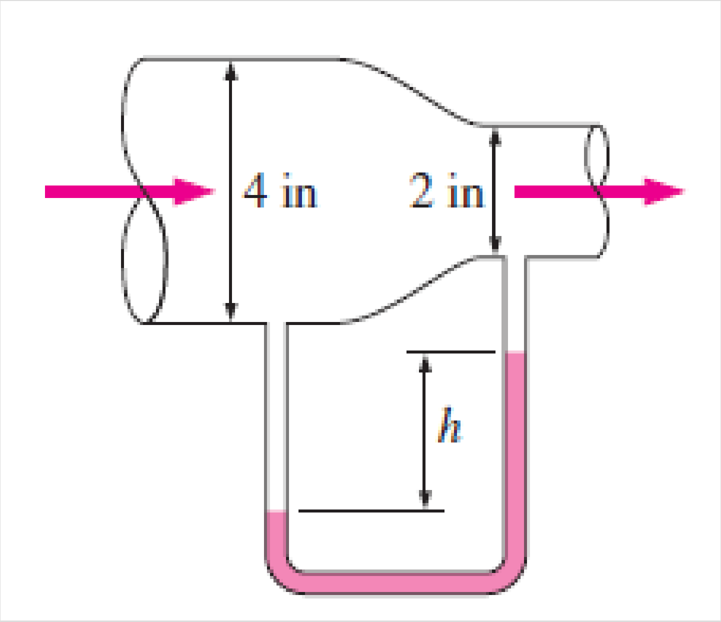
Example:

A well-insulated rigid tank contains 5 kg of a saturated liquid–vapor mixture of water at 100 kPa. Initially, three-quarters of the mass is in the liquid phase. An electric resistor placed in the tank is connected to a 110-V source, and a current of 8 A flows through the resistor when the switch is turned on. Determine how long it will take to vaporize all the liquid in the tank. Also, show the process on a *T*-*v* diagram with respect to saturation lines.



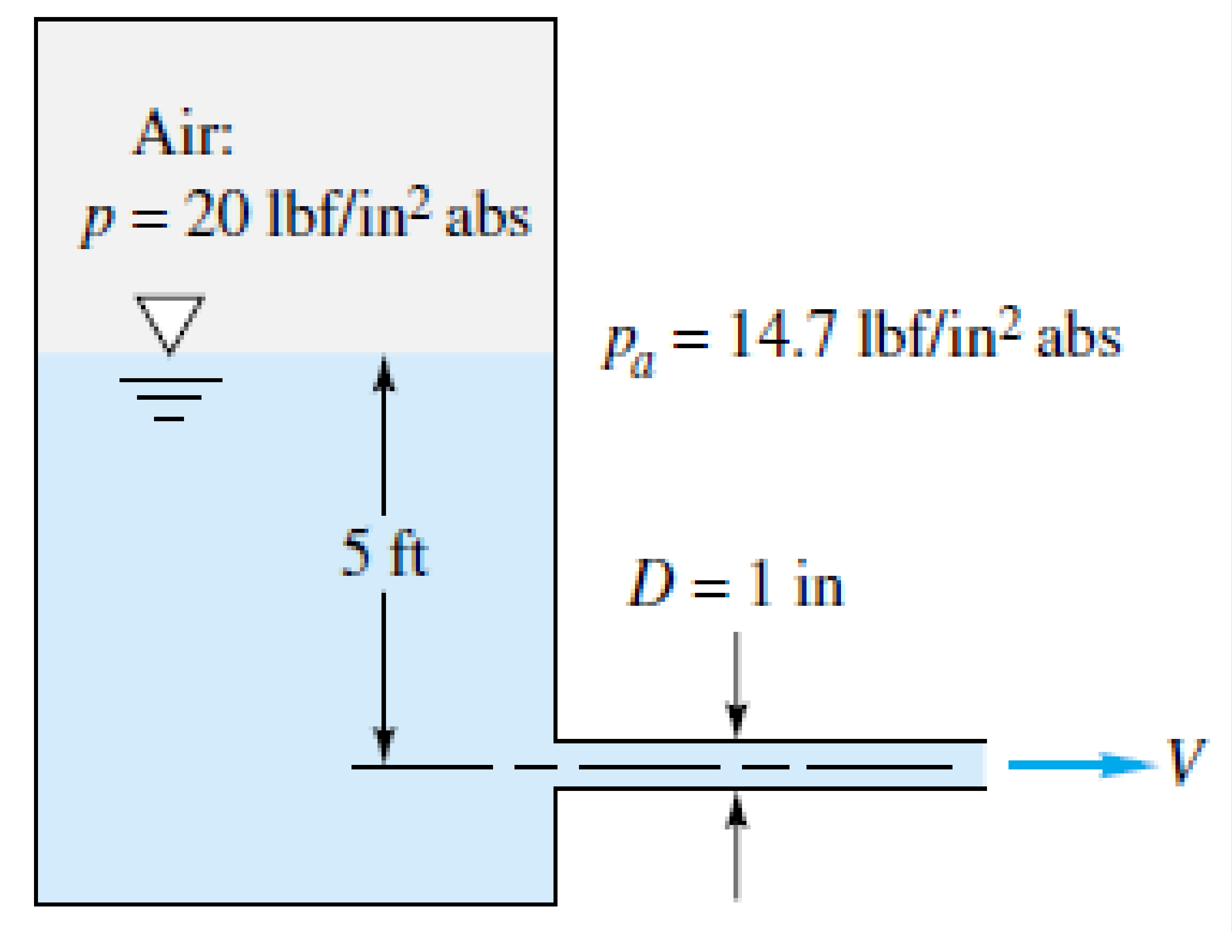
Example:

Water flows through a horizontal pipe at a rate of 1 gal/s. The pipe consist of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting frictional effects, determine the differential height of mercury between the two pipe sections.



Example:

The liquid in the Figure has a s = 0,85. Estimate the flow rate from the tank for *a*) no losses and *b*) pipe losses $h_L = 4.5\ V^2/(2g)$.



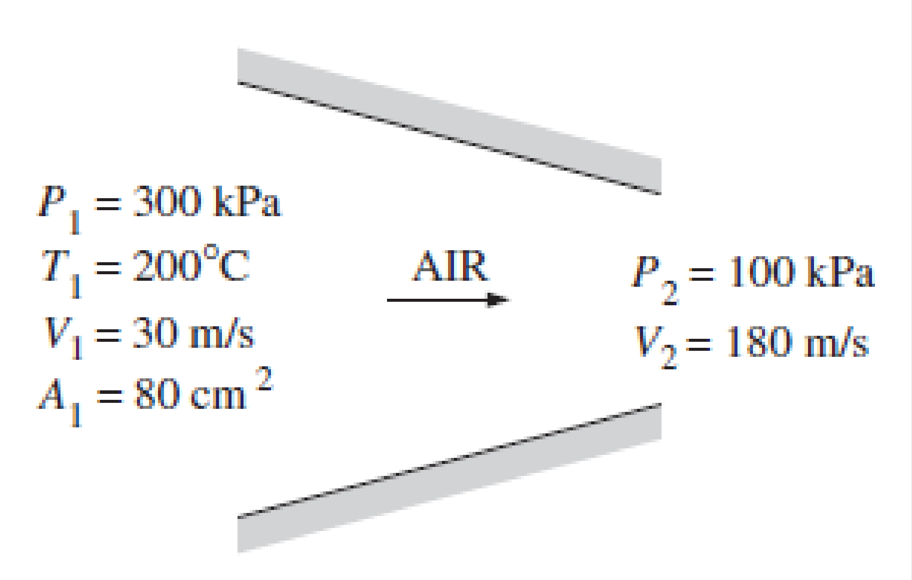
Example:

Air enters an adiabatic nozzle steadily at 300 kPa, 200°C, and 30 m/s and leaves at 100 kPa and 180 m/s. The inlet area of the nozzle is 80 cm2. Determine:

(a) the mass flow rate through the nozzle,

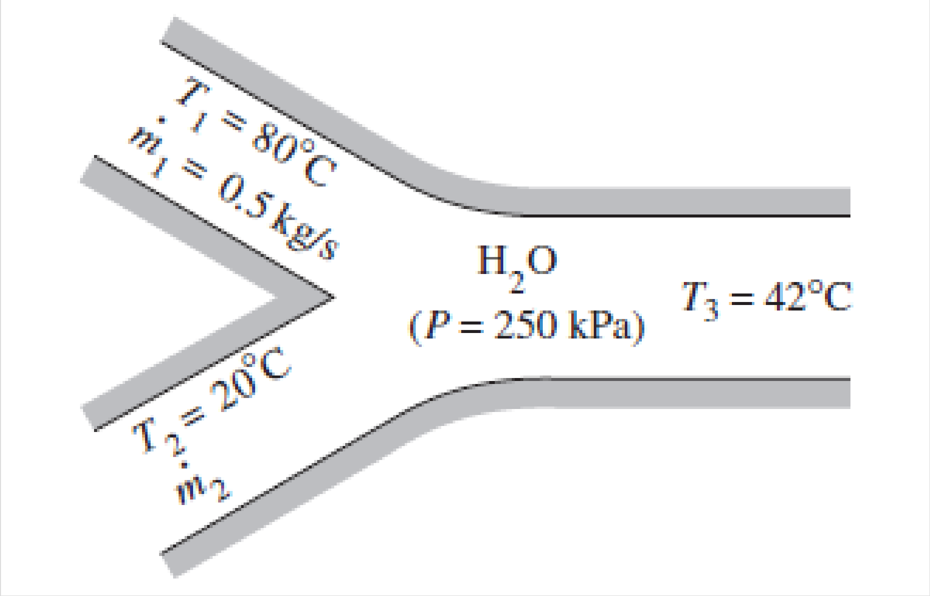
(b) the exit temperature of the air, and

(c) the exit area of the nozzle



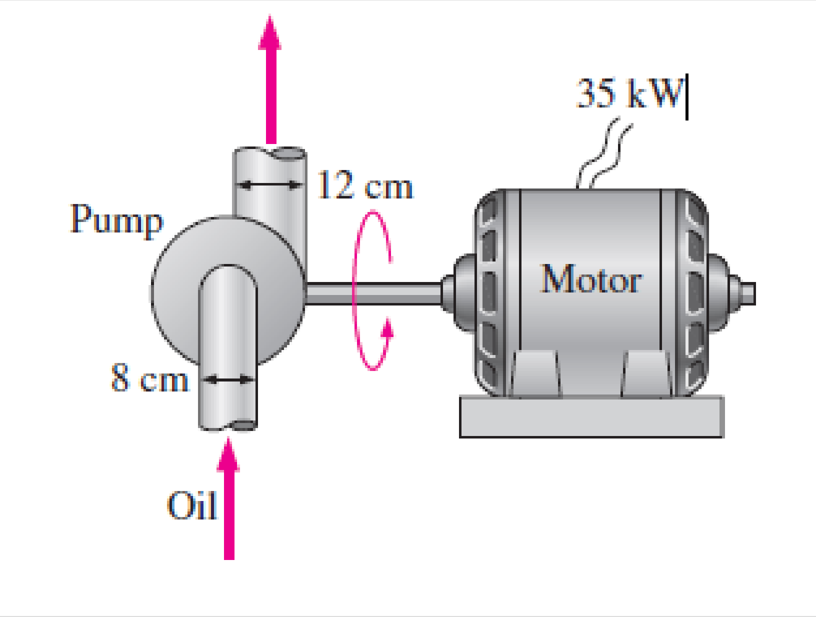
Example:

A hot-water stream at 80°C enters a mixing chamber with a mass flow rate of 0.5 kg/s where it is mixed with a stream of cold water at 20°C. If it is desired that the mixture leave the chamber at 42°C, determine the mass flow rate of the cold-water stream. Assume all the streams are at a pressure of 250 kPa.



Example:

An oil pump is drawing 35 kW of electric power while pumping oil with $\rho=860\ kg/m^3$ at a rate of $0.15 m^3/s$. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 450kPa and the motor efficiency is 80%, determine the mechanical efficiency of the pump.



MOMENTUM ANALYSIS

NEWTON'S LAWS

Newton's laws: Relations between motions of bodies and the forces acting on them.

Newton's first law: A body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.

Therefore, a body tends to preserve its state of inertia.

Newton's second law: The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.

Newton's third law: When a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

Therefore, the direction of an exposed reaction force depends on the body taken as the system.

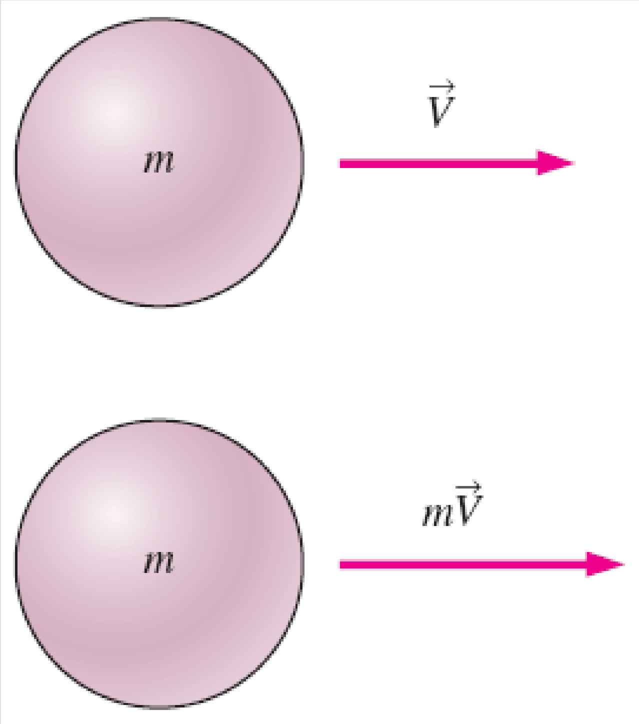
Newton's second law:

$$\vec{F} = m\vec{a} = m\,\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt}$$

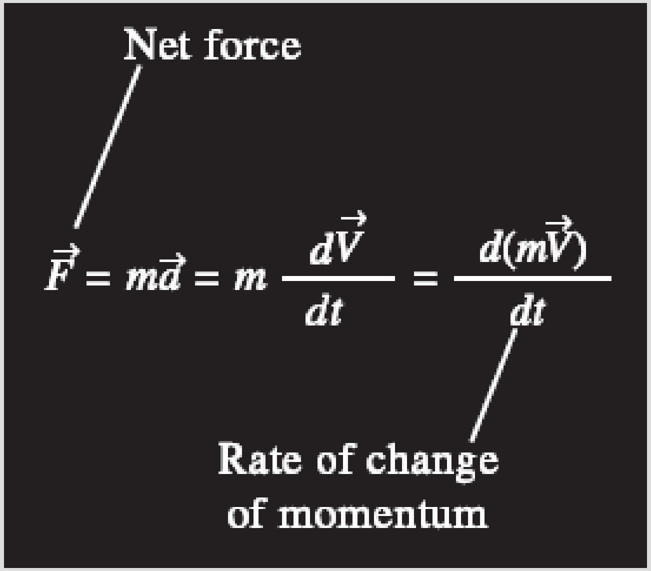
Linear momentum or just the *momentum* of the body: The product of the mass and the velocity of a body.

Newton's second law is usually referred to as the *linear momentum equation*.

Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.



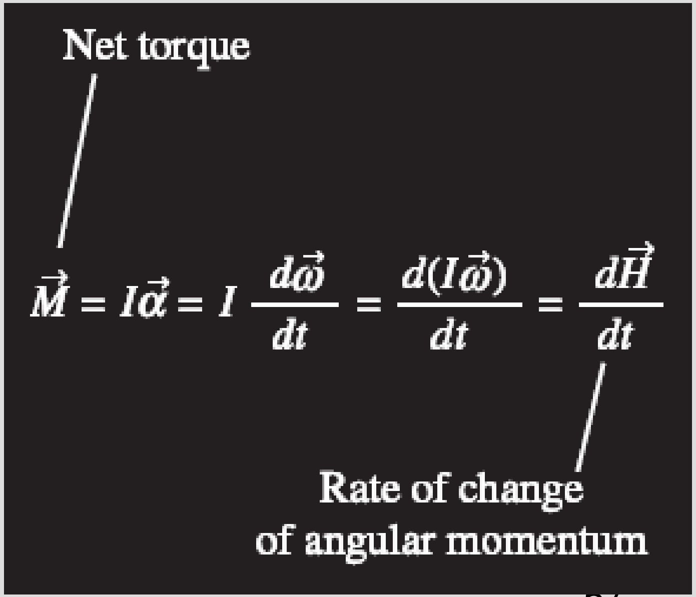
Newton's second law is also expressed as *the rate of change of the momentum of a body is equal to the net force acting on it*.

The counterpart of Newton's second law for rotating rigid bodies is expressed as $\vec{M} = I\vec{\alpha}$, where $\vec{M}$ is the net moment or torque applied on the body, I is the moment of inertia of the body about the axis of rotation, and $\vec{\alpha}$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $d\vec{H}/dt$ as

Angular momentum equation:
$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt} \qquad \textbf{(13–2)}$$

Angular momentum about x-axis:
$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

The conservation of angular momentum Principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.



The rate of change of the angular momentum of a body is equal to the net torque acting on it.

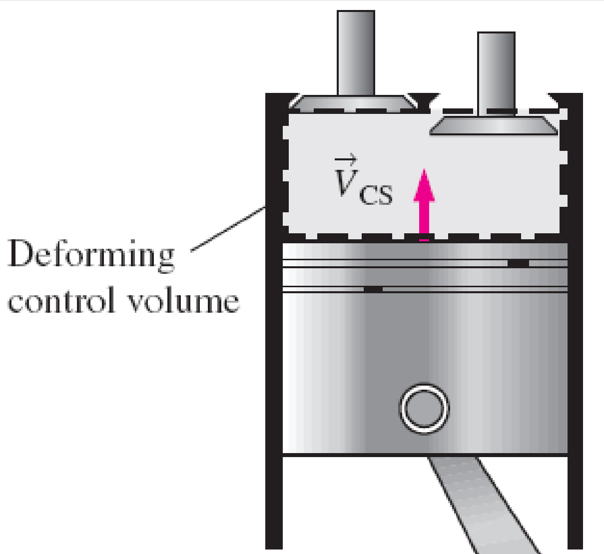
CHOOSING A CONTROL VOLUME

A control volume can be selected as any arbitrary region in space through which fluid flows, and its bounding control surface can be fixed, moving, and even deforming during flow.

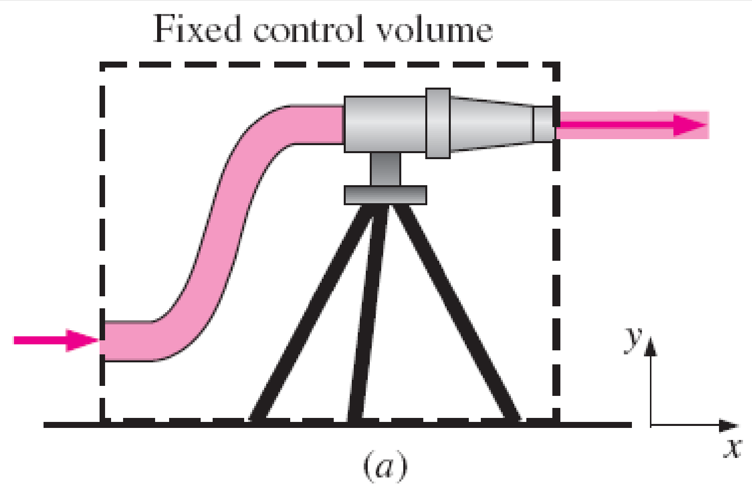
Many flow systems involve stationary hardware firmly fixed to a stationary surface, and such systems are best analyzed using *fixed* control volumes.

When analyzing flow systems that are moving or deforming, it is usually more convenient to allow the control volume to *move* or *deform*.

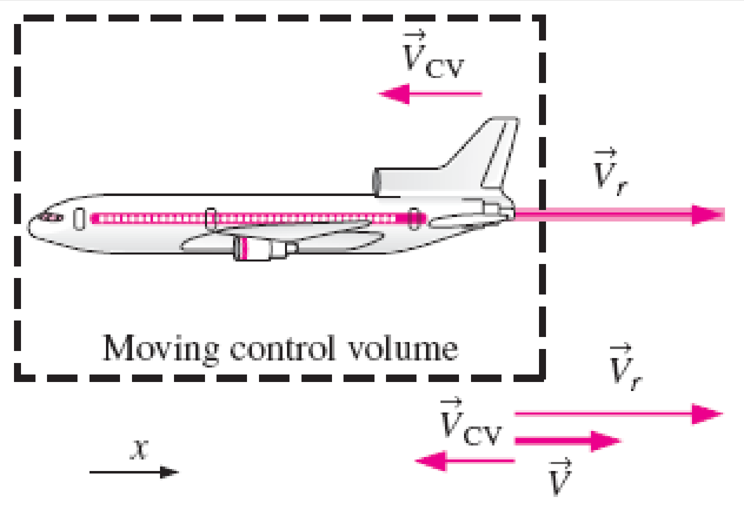
In *deforming* control volume, part of the control surface moves relative to other parts.



(c)



(a)



FORCES ACTING ON A CONTROL VOLUME

The forces acting on a control volume consist of

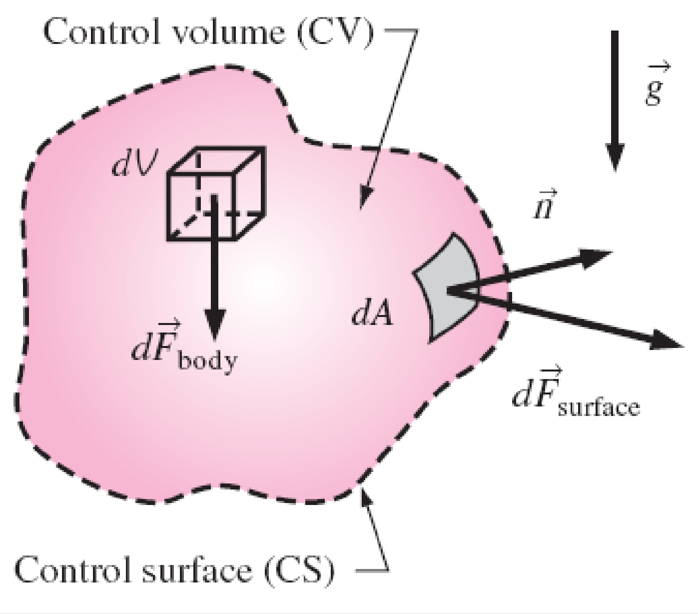
body forces that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and

surface forces that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact).

Only external forces are considered in the analysis.

Total force acting on control volume:

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$$



The total force acting on a control volume is composed of body forces and surface forces; body force is shown on a differential volume element, and surface force is shown on a differential surface element.

The most common body force is that of gravity, which exerts a downward force on every differential element of the control volume.

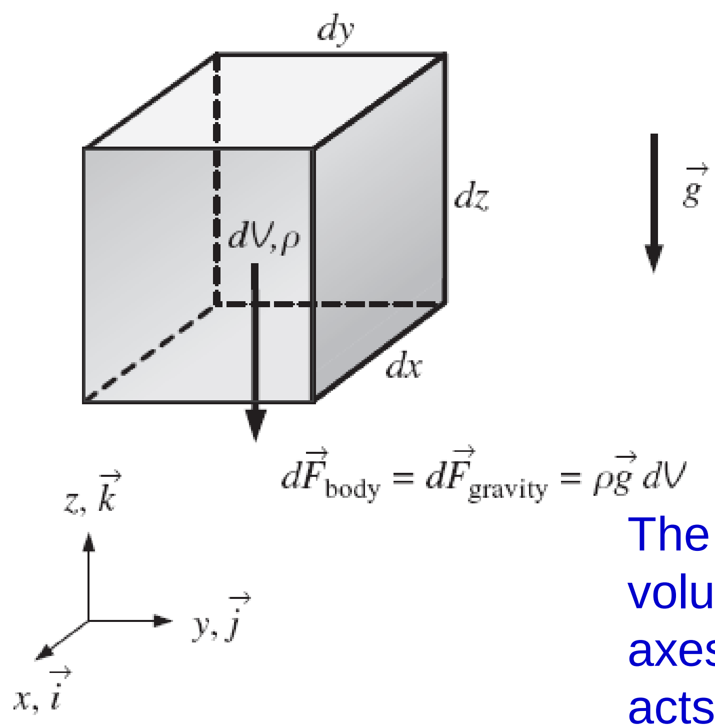
Gravitational force acting on a fluid element: $d\vec{F}_{\text{gravity}} = \rho \vec{g}\, dV$

Gravitational vector in Cartesian coordinates: $\vec{g} = -g\vec{k}$

Total body force acting on control volume: $\sum \vec{F}_{\text{body}} = \int_{\text{CV}} \rho \vec{g}\, dV = m_{\text{CV}}\vec{g}$

Total force:

$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$$



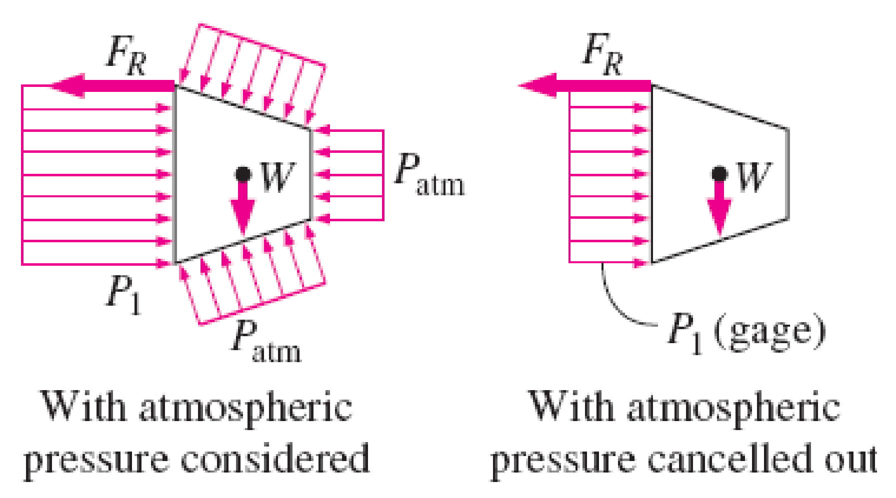
Surface forces are not as simple to analyze since they consist of both *normal* and *tangential* components.
Normal stresses are composed of pressure (which always acts inwardly normal) and viscous stresses.
Shear stresses are composed entirely of viscous stresses.

The gravitational force acting on a differential volume element of fluid is equal to its weight; the axes have been rotated so that the gravity vector acts *downward* in the negative *z*-direction.

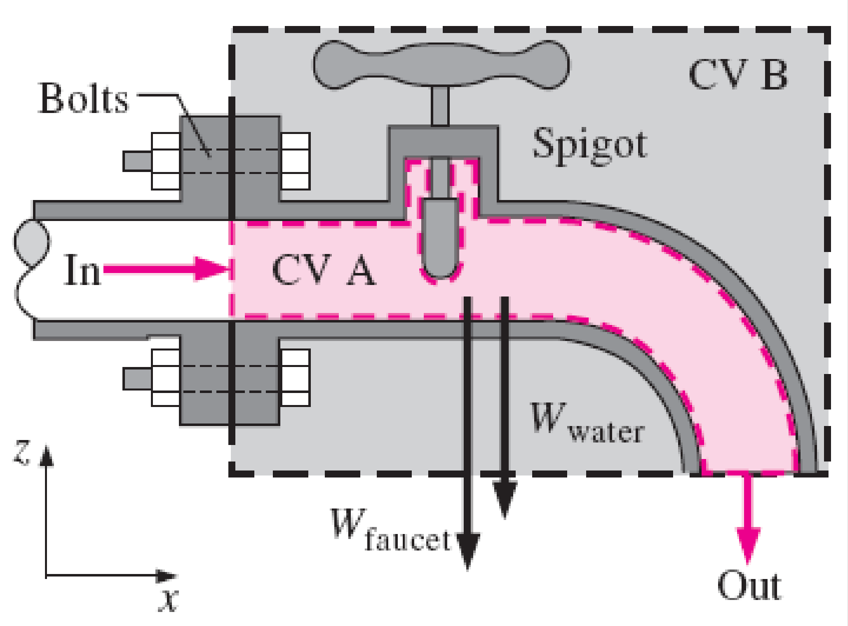
A common simplification in the application of Newton's laws of motion is to subtract the *atmospheric pressure* and work with gage pressures.

This is because atmospheric pressure acts in all directions, and its effect cancels out in every direction.

This means we can also ignore the pressure forces at outlet sections where the fluid is discharged to the atmosphere since the discharge pressure in such cases is very near atmospheric pressure at subsonic velocities.



Atmospheric pressure acts in all directions, and thus it can be ignored when performing force balances since its effect cancels out in every direction.



Cross section through a faucet assembly, illustrating the importance of choosing a control volume wisely; CV B is much easier to work with than CV A.

THE LINEAR MOMENTUM EQUATION

Newton's second law for a system of mass m subjected to a net force $\vec{F}$ is expressed from Eq. 14–1 as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V}) \quad \text{(13–9)}$$

where $m\vec{V}$ is the **linear momentum** of the system. Noting that both the density and velocity may change from point to point within the system, Newton's second law can be expressed more generally as

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{sys}} \rho\vec{V}\, dV \quad \text{(13–10)}$$

where $\rho\vec{V}\, dV$ is the momentum of a differential element dV, which has mass $\delta m = \rho dV$.

Newton's second law can be stated as *the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system*. This statement is valid for a coordinate system that is at rest or moves with a constant velocity, called an *inertial coordinate system* or *inertial reference frame*.

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\vec{V}\,dV + \int_{CS} \rho\vec{V}\,(\vec{V}_r \cdot \vec{n})\,dA$$

General: $$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\,dV + \int_{CS} \rho\vec{V}(\vec{V}_r \cdot \vec{n})\,dA$$

$$\vec{V}_r = \vec{V} - \vec{V}_{CS}$$

(The sum of all external forces acting on a CV) = (The time rate of change of the linear momentum of the contents of the CV) + (The net flow rate of linear momentum out of the control surface by mass flow)

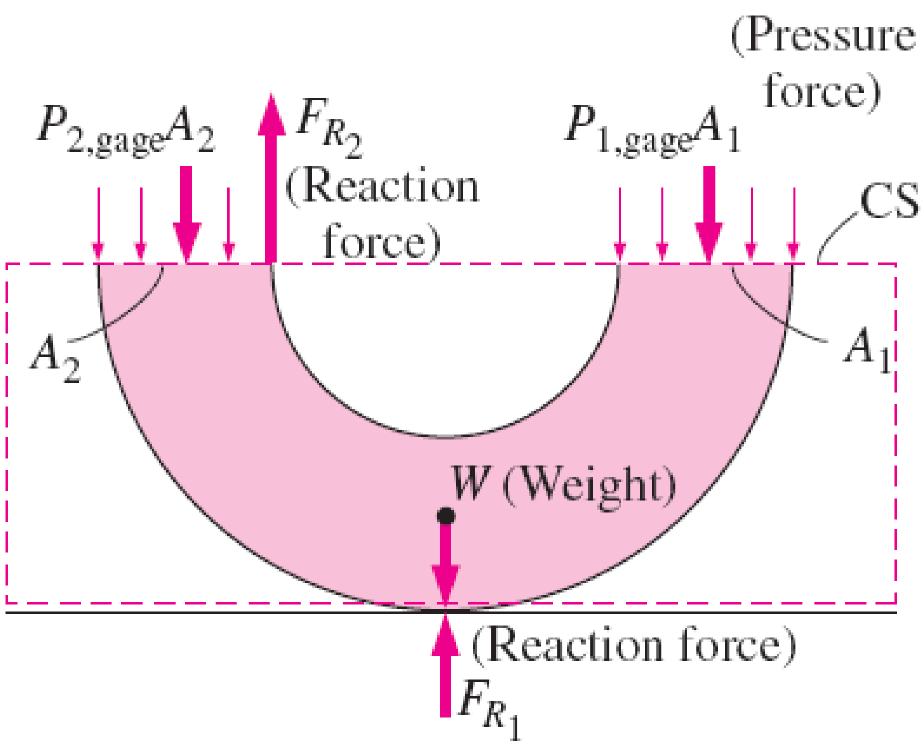
Fixed CV: $$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\,dV + \int_{CS} \rho\vec{V}(\vec{V} \cdot \vec{n})\,dA$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b\,dV + \int_{CS} \rho b(\vec{V}_r \cdot \vec{n})\,dA$$

$B = m\vec{V}$ $\qquad$ $b = \vec{V}$ $\qquad$ $b = \vec{V}$

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\vec{V}\,dV + \int_{CS} \rho\vec{V}(\vec{V}_r \cdot \vec{n})\,dA$$

The linear momentum equation is obtained by replacing B in the Reynolds transport theorem by the momentum $m\vec{V}$, and b by the momentum per unit mass $\vec{V}$.



An 180° elbow supported by the ground

The momentum equation is commonly used to calculate the forces (usually on support systems or connectors) induced by the flow.

In most flow systems, the sum of forces $\Sigma\vec{F}$ consists of weights, pressure forces, and reaction forces. Gage pressures are used here since atmospheric pressure cancels out on all sides of the control surface.

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, dV + \int_{CS} \rho\vec{V}(\vec{V}\cdot\vec{n})\, dA$$

Special Cases

$$\sum \vec{F} = \int_{CS} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) dA$$

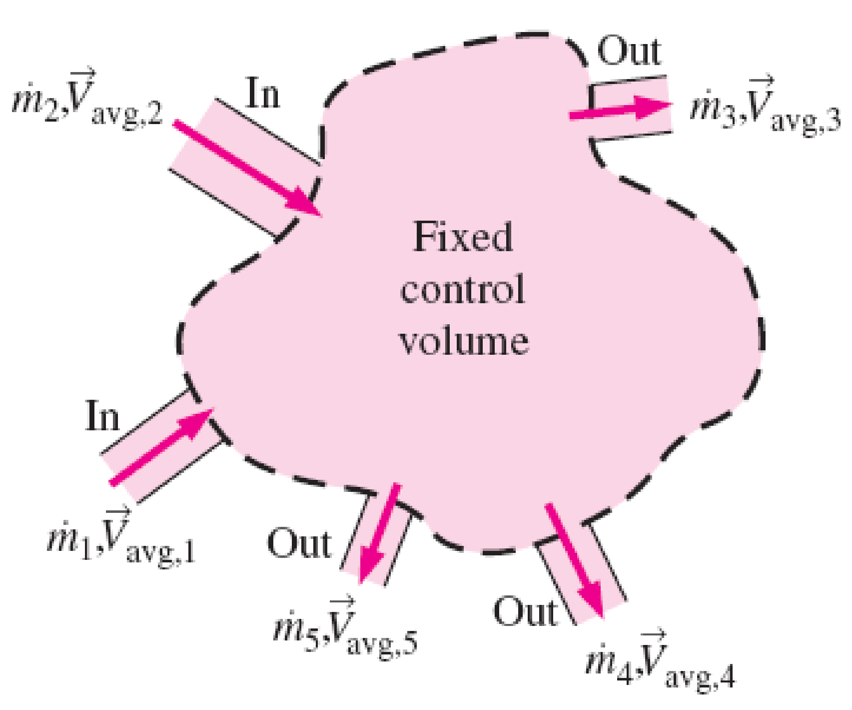
Steady flow

$$\dot{m} = \int_{A_c} \rho(\vec{V} \cdot \vec{n}) dA_c = \rho V_{avg} A_c$$

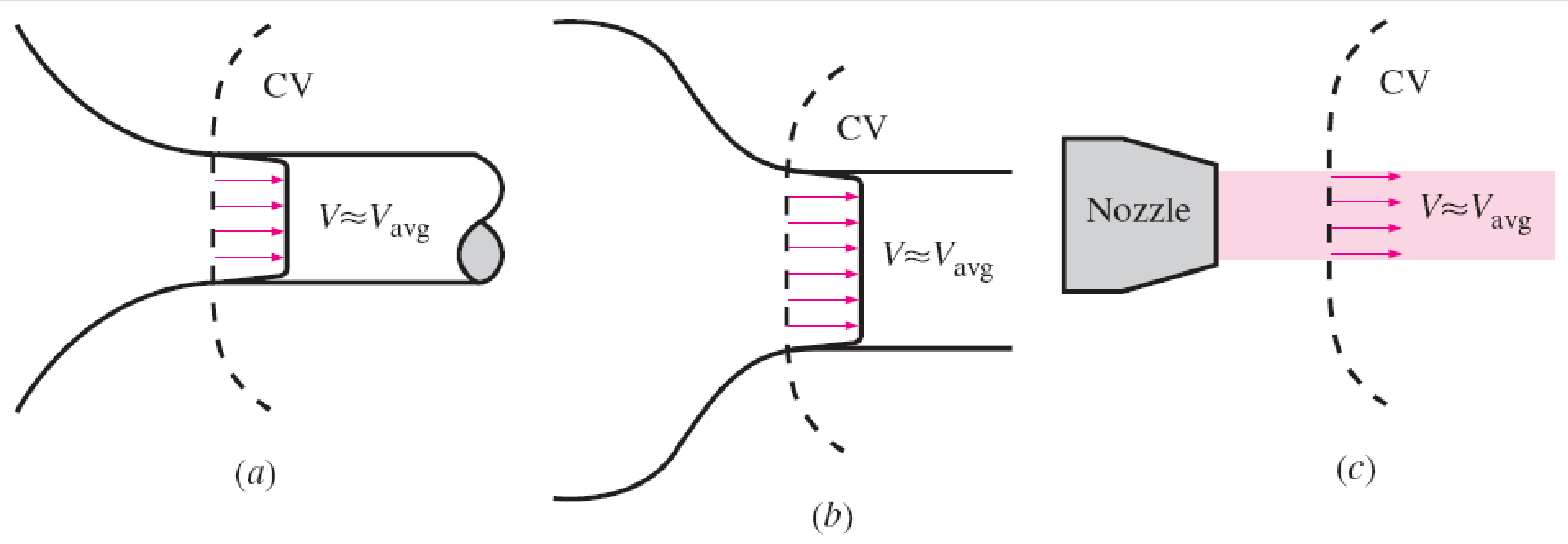
Mass flow rate across an inlet or outlet

$$\int_{A_c} \rho \vec{V}(\vec{V} \cdot \vec{n}) dA_c = \rho V_{avg} A_c \vec{V}_{avg} = \dot{m} \vec{V}_{avg}$$

Momentum flow rate across a uniform inlet or outlet:



In a typical engineering problem, the control volume may contain many inlets and outlets; at each inlet or outlet we define the mass flow rate and the average velocity.



Examples of inlets or outlets in which the uniform flow approximation is reasonable:

(*a*) the well-rounded entrance to a pipe,

(*b*) the entrance to a wind tunnel test section, and

(*c*) a slice through a free water jet in air.

Momentum-Flux Correction Factor, β

The velocity across most inlets and outlets is *not* uniform.

The control surface integral of Eq. 13–13 may be converted into algebraic form using a dimensionless correction factor β, called the momentum-flux correction factor.

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA \qquad (13\text{-}13)$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \sum_{out} \beta \dot{m} \vec{V}_{avg} - \sum_{in} \beta \dot{m} \vec{V}_{avg}$$

Momentum flux across an inlet or outlet: $\int_{A_c} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA_c = \beta \dot{m} \vec{V}_{avg}$

$$\beta = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\dot{m} V_{avg}} = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\rho V_{avg} A_c V_{avg}}$$

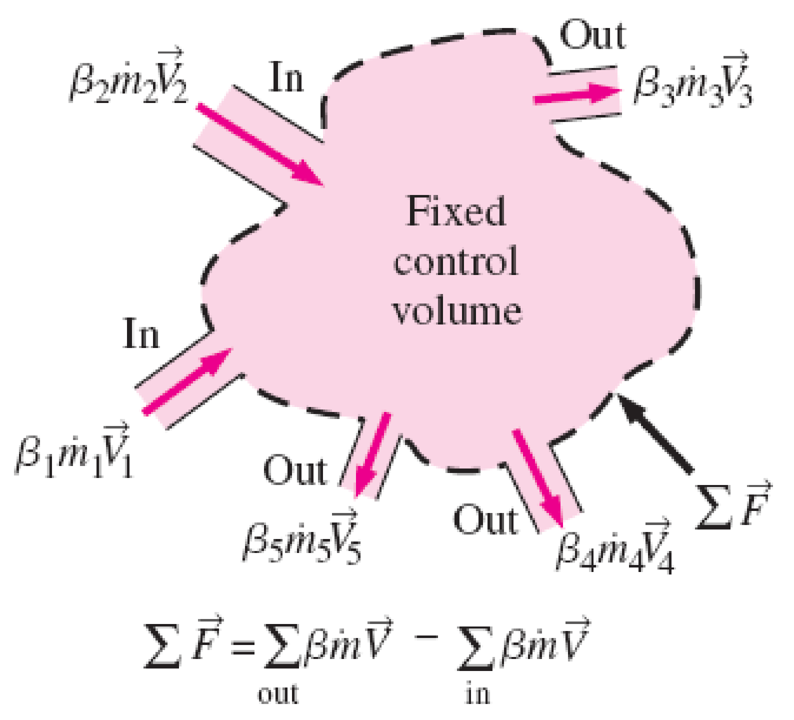
β is always greater than or equal to 1.
β is close to 1 for turbulent flow and not very close to 1 for fully developed laminar flow.

Momentum-flux correction factor: $\beta = \frac{1}{A_c}\int_{A_c} \left(\frac{V}{V_{avg}}\right)^2 dA_c$

Steady Flow

Steady linear momentum equation: $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$

The net force acting on the control volume during steady flow is equal to the difference between the rates of outgoing and incoming momentum flows.



The net force acting on the control volume during steady flow is equal to the difference between the outgoing and the incoming momentum fluxes.

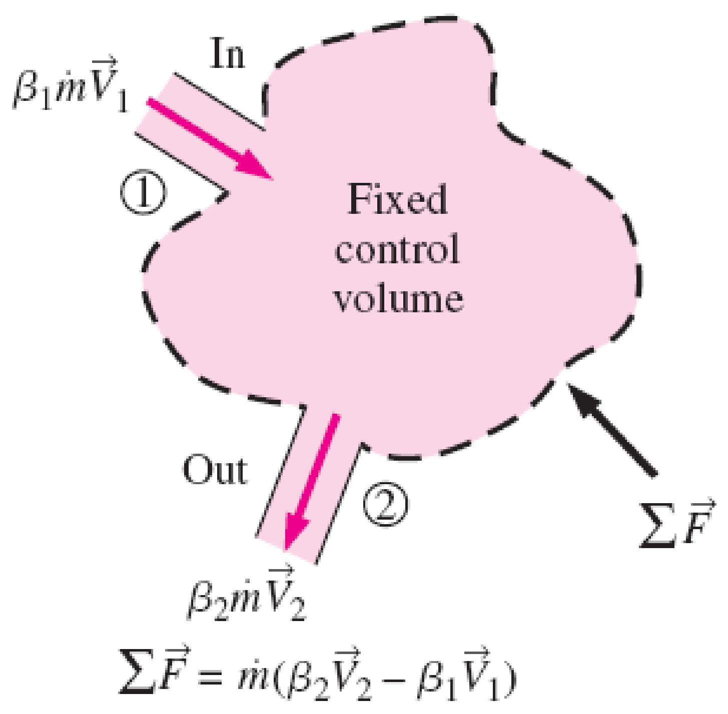
Steady Flow with One Inlet and One Outlet

$$\sum \vec{F} = \dot{m}\,(\beta_2\vec{V}_2 - \beta_1\vec{V}_1)$$

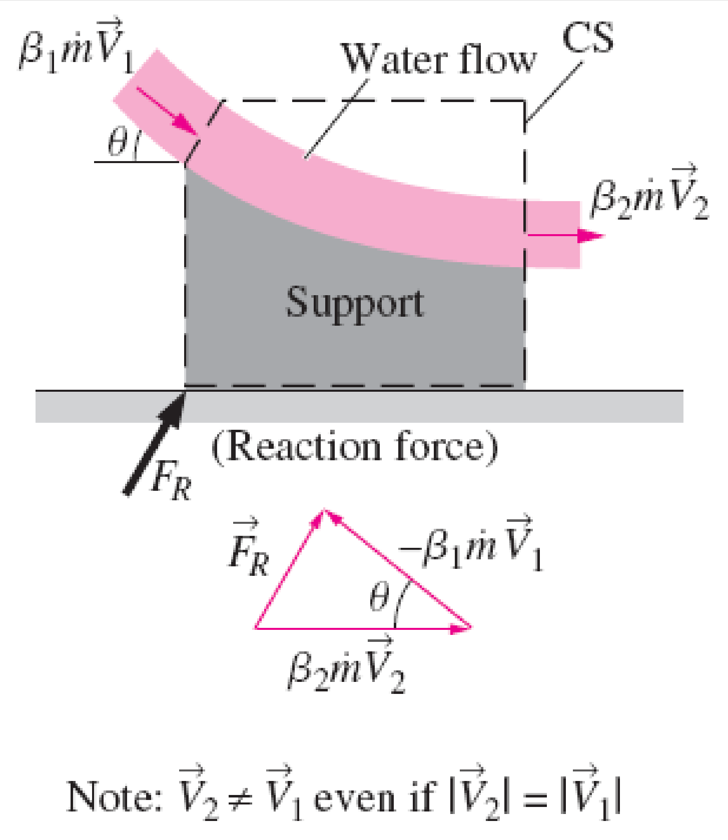
One inlet and one outlet

$$\sum F_x = \dot{m}(\beta_2 V_{2,x} - \beta_1 V_{1,x})$$

Along x-coordinate



A control volume with only one inlet and one outlet.



The determination by vector addition of the reaction force on the support caused by a change of direction of water.

Flow with No External Forces

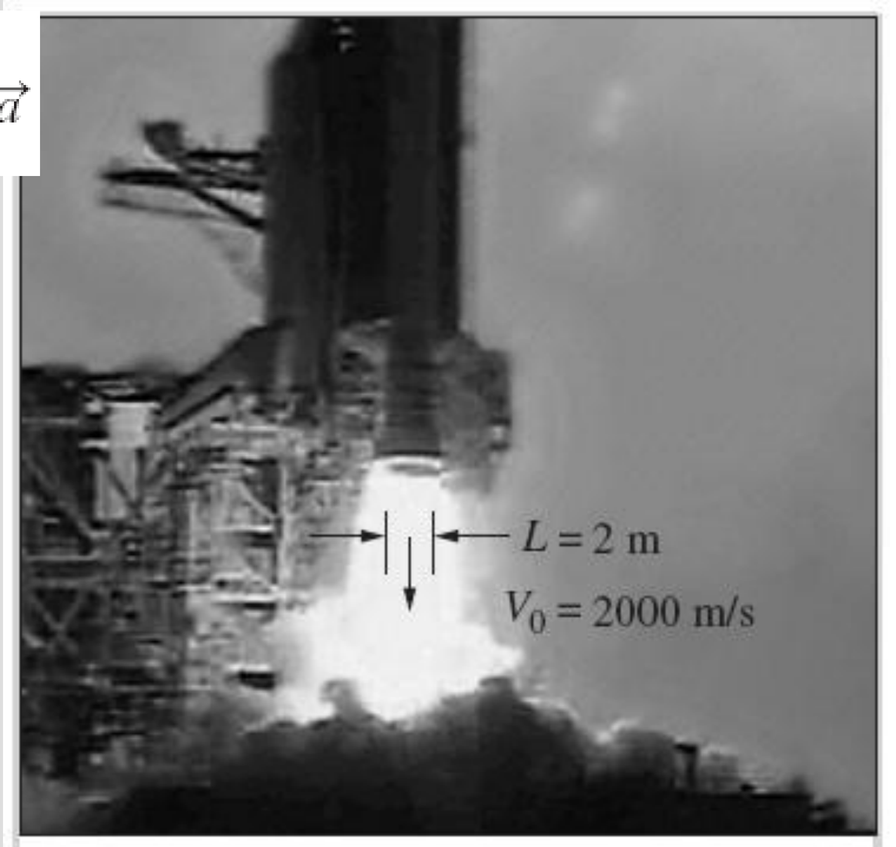
$$\text{No external forces:} \qquad 0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

In the absence of external forces, the rate of change of the momentum of a control volume is equal to the difference between the rates of incoming and outgoing momentum flow rates.

$$\frac{d(m\vec{V})_{CV}}{dt} = m_{CV}\frac{d\vec{V}_{CV}}{dt} = (m\vec{a})_{CV} = m_{CV}\vec{a}$$

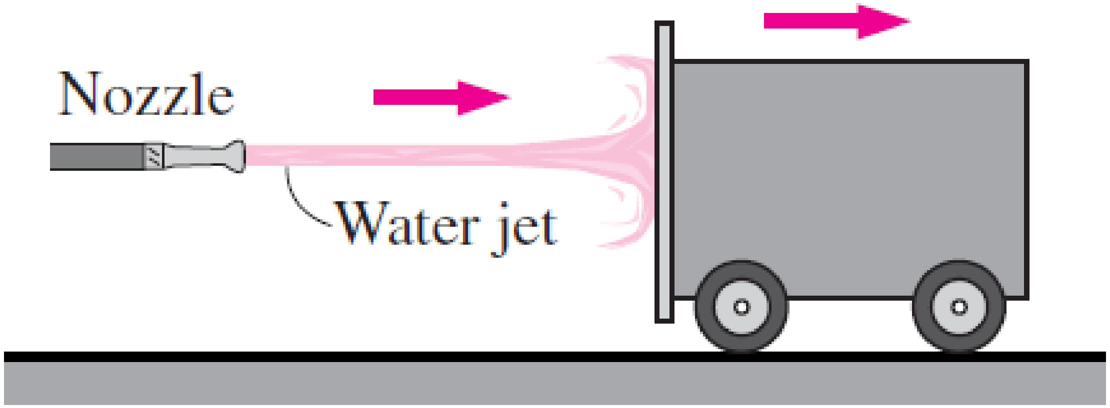
$$\vec{F}_{thrust} = m_{CV}\vec{a} = \sum_{in} \beta \dot{m} \vec{V} - \sum_{out} \beta \dot{m} \vec{V}$$

The thrust needed to lift the space shuttle is generated by the rocket engines as a result of momentum change of the fuel as it is accelerated from about zero to an exit speed of about 2000 m/s after combustion.



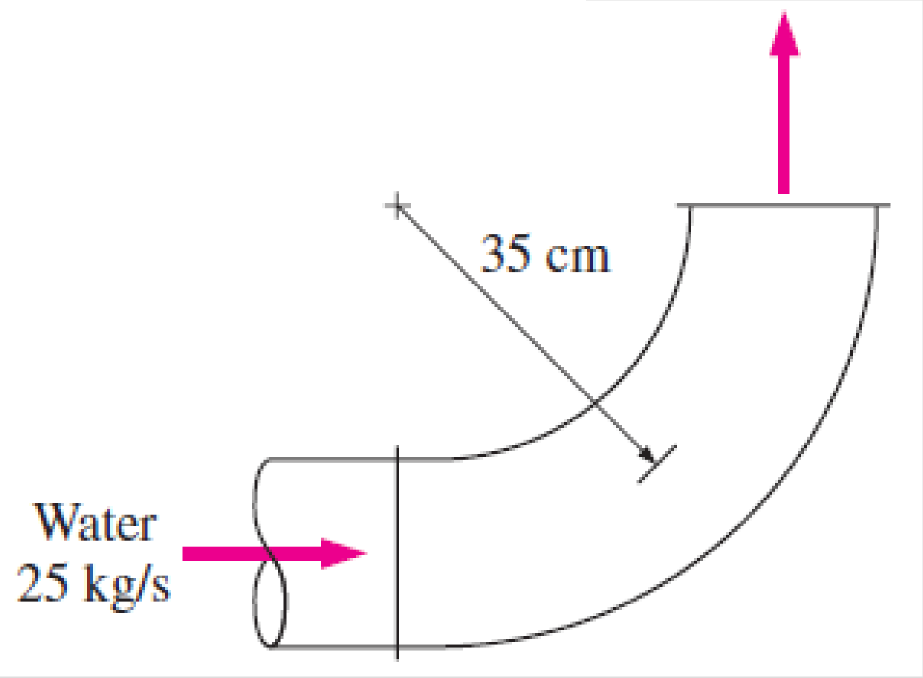
Example:

A constant-velocity horizontal water jet from a stationary nozzle impinges normally on a vertical flat plate that is held in a nearly frictionless track. As the water jet hits the plate, it begins to move due to the water force. Will the acceleration of the plate remain constant or change? Explain.



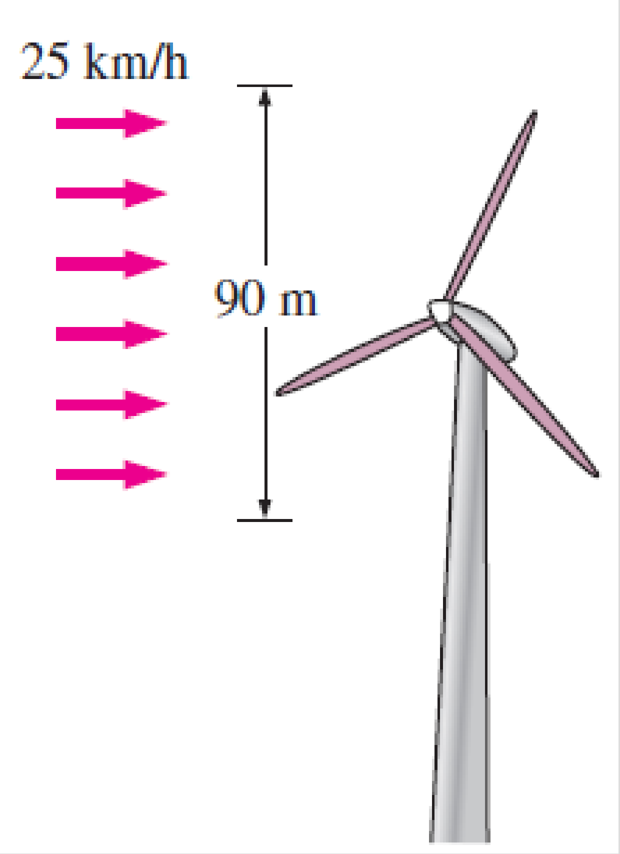
Example:

A 90° elbow is used to direct water flow at a rate of 25 kg/s in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.



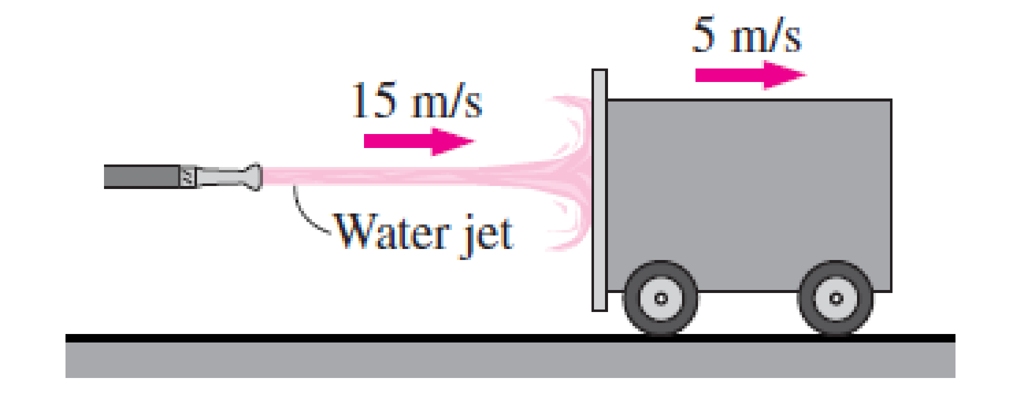
Example:

Commercially available large wind turbines have blade span diameters as large as 100 m and generate over 3 MW of electric power at peak design conditions. Consider a wind turbine with a 90-m blade span subjected to 25-km/h steady winds. If the combined turbine–generator efficiency of the wind turbine is 32 percent, determine (*a*) the power generated by the turbine and (*b*) the horizontal force exerted by the wind on the supporting mast of the turbine. Take the density of air to be 1.25 kg/m^3, and disregard frictional effects.



Example:

Water accelerated by a nozzle to 15 m/s strikes the vertical back surface of a cart moving horizontally at a constant velocity of 5 m/s in the flow direction. The mass flow rate of water is 25 kg/s. After the strike, the water stream splatters off in all directions in the plane of the back surface. (*a*) Determine the force that needs to be applied on the brakes of the cart to prevent it from accelerating. (*b*) If this force were used to generate power instead of wasting it on the brakes, determine the maximum amount of power that can be generated.



REVIEW OF ROTATIONAL MOTION AND ANGULAR MOMENTUM

Rotational motion: A motion during which all points in the body move in circles about the axis of rotation.

Rotational motion is described with angular quantities such as the angular distance θ, angular velocity ω, and angular acceleration α.

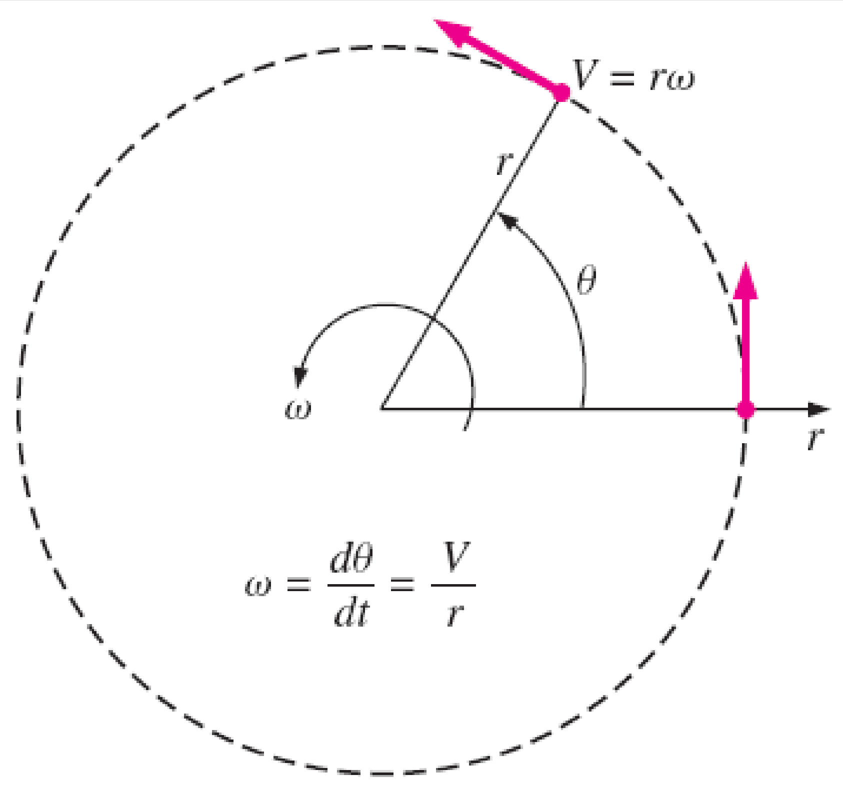
Angular velocity: The angular distance traveled per unit time.

Angular acceleration: The rate of change of angular velocity.

$$\omega = \frac{d\theta}{dt} = \frac{d(l/r)}{dt} = \frac{1}{r}\frac{dl}{dt} = \frac{V}{r}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{dV}{dt} = \frac{a_t}{r}$$

$$V = r\omega \qquad \text{and} \qquad a_t = r\alpha$$



The relations between angular distance θ, angular velocity ω, and linear velocity V.

- Newton's second law requires that there must be a force acting in the tangential direction to cause angular acceleration.
- The strength of the rotating effect, called the *moment or torque*, is proportional to the magnitude of the force and its distance from the axis of rotation.
- The perpendicular distance from the axis of rotation to the line of action of the force is called the *moment arm*, and the torque *M* acting on a point mass *m* at a normal distance *r* from the axis of rotation is expressed as

$$M = rF_t = rma_t = mr^2\alpha$$

Torque

$$M = \int_{\text{mass}} r^2\alpha \, \delta m = \left[\int_{\text{mass}} r^2 \, \delta m\right]\alpha = I\alpha$$

I is the *moment of inertia* of the body about the axis of rotation, which is a measure of the inertia of a body against rotation.

Unlike mass, the rotational inertia of a body also depends on the distribution of the mass of the body with respect to the axis of rotation.

$$\text{Mass, } m \leftrightarrow \text{Moment of inertia, } I$$
$$\text{Linear acceleration, } a \leftrightarrow \text{Angular acceleration, } \alpha$$
$$\text{Linear velocity, } V \leftrightarrow \text{Angular velocity, } \omega$$
$$\text{Linear momentum} \leftrightarrow \text{Angular momentum}$$
$$m\vec{V} \leftrightarrow I\vec{\omega}$$

$$\text{Force, } F \leftrightarrow \text{Torque, } M$$
$$\vec{F} = m\vec{a} \leftrightarrow \vec{M} = I\vec{\alpha}$$
$$\text{Moment of force, } M \leftrightarrow \text{Moment of momentum, } H$$
$$\vec{M} = \vec{r} \times \vec{F} \leftrightarrow \vec{H} = \vec{r} \times m\vec{V}$$

Analogy between corresponding linear and angular quantities.

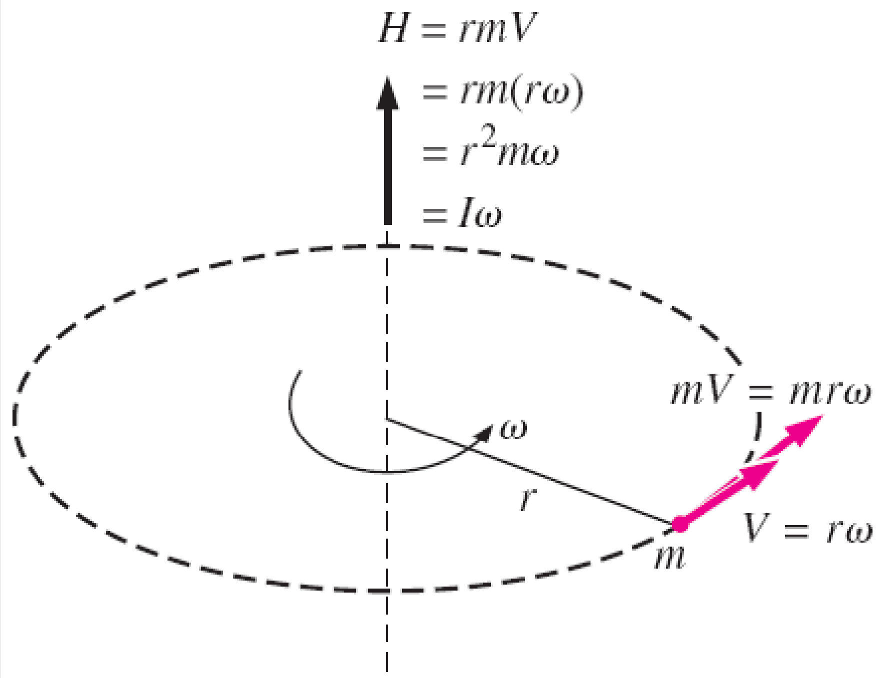
$$H = \int_{\text{mass}} r^2\omega\, \delta m = \left[\int_{\text{mass}} r^2\, \delta m\right]\omega = I\omega$$

Angular momentum

$$\vec{H} = I\vec{\omega}$$

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

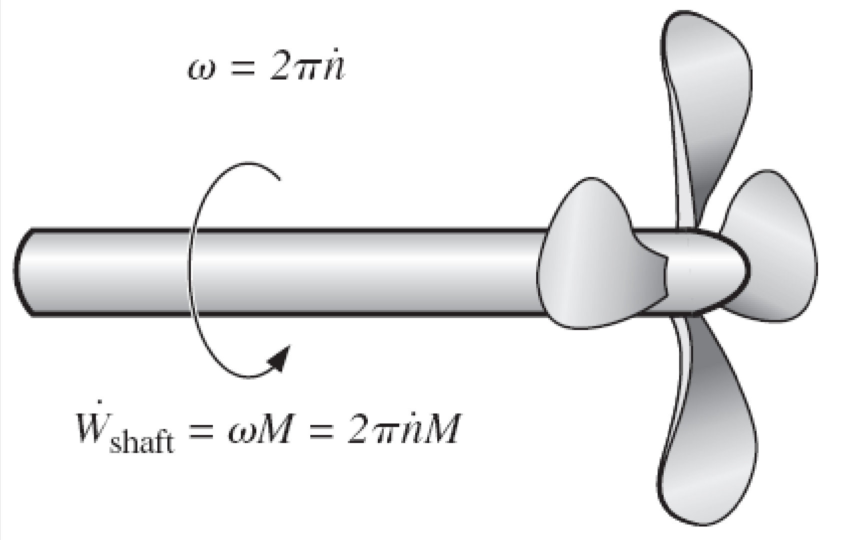
Angular momentum equation



Angular momentum of point mass *m* rotating at angular velocity ω at distance *r* from the axis of rotation.

$$\omega = \frac{2\pi\dot{n}}{60} \quad \text{(rad/s)}$$

Angular velocity versus rpm



The relations between angular velocity, rpm, and the power transmitted through a shaft.

$$\dot{W}_{\text{shaft}} = FV = Fr\omega = M\omega$$

$$\dot{W}_{\text{shaft}} = \omega M = 2\pi \dot{n} M \qquad \text{(W)}$$ Shaft power

$$\text{KE}_r = \tfrac{1}{2} I\omega^2$$ Rotational kinetic energy

During rotational motion, the direction of velocity changes even when its magnitude remains constant. Velocity is a vector quantity, and thus a change in direction constitutes a change in velocity with time, and thus acceleration. This is called centripetal acceleration.

$$a_r = \frac{V^2}{r} = r\omega^2$$

Centripetal acceleration is directed toward the axis of rotation (opposite direction of radial acceleration), and thus the radial acceleration is negative. Centripetal acceleration is the result of a force acting on an element of the body toward the axis of rotation, known as the centripetal force, whose magnitude is $F_r = mV^2/r$.

Tangential and radial accelerations are perpendicular to each other, and the total linear acceleration is determined by their vector sum:

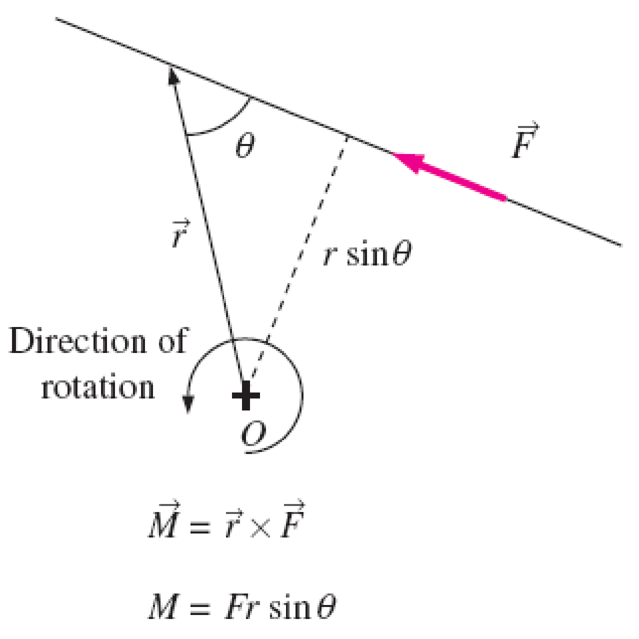
$$\vec{a} = \vec{a}_t + \vec{a}_r$$

THE ANGULAR MOMENTUM EQUATION

Many engineering problems involve the moment of the linear momentum of flow streams, and the rotational effects caused by them.

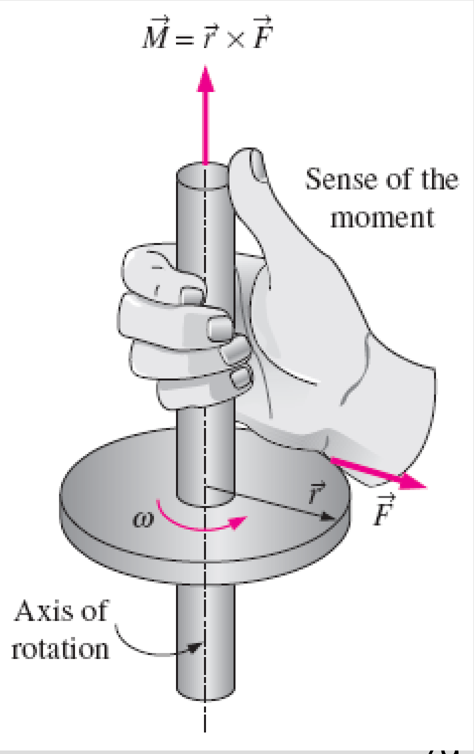
Such problems are best analyzed by the *angular momentum equation*, also called the *moment of momentum equation.*

An important class of fluid devices, called *turbomachines*, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.



The moment of a force $\vec{F}$ about a point O is the vector product of the position vector $\vec{r}$ and $\vec{F}$.

A force whose line of action passes through point *O* produces zero moment about point *O*.



The determination of the direction of the moment by the right-hand rule.

Moment of momentum

$$\vec{H} = \vec{r} \times m\vec{V}$$

Moment of momentum (system)

$$\vec{H}_{sys} = \int_{sys} (\vec{r} \times \vec{V})\rho \, dV$$

$$\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt}\int_{sys} (\vec{r} \times \vec{V})\rho \, dV$$

Rate of change of moment of momentum

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b(\vec{V}_r \cdot \vec{n})\, dA$$

$B = \vec{H}$ $\quad b = \vec{r} \times \vec{V}$ $\quad b = \vec{r} \times \vec{V}$

$$\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

The angular momentum equation is obtained by replacing B in the Reynolds transport theorem by the angular momentum $\vec{H}$, and b by the angular momentum per unit mass $\vec{r} \times \vec{V}$.

Angular momentum equation for a system

$$\sum \vec{M} = \frac{d\vec{H}_{sys}}{dt}$$

$$\sum \vec{M} = \sum (\vec{r} \times \vec{F})$$

$$\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

General:
$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

(The sum of all external moments acting on a CV) = (The time rate of change of the angular momentum of the contents of the CV) + (The net flow rate of angular momentum out of the control surface by mass flow)

Fixed CV:
$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n})\, dA$$

Special Cases

During *steady flow*, the amount of angular momentum within the control volume remains constant, and thus the time rate of change of angular momentum of the contents of the control volume is zero.

$$\text{Steady flow:} \qquad \sum \vec{M} = \int_{CS} (\vec{r} \times \vec{V}) \rho (\vec{V}_r \cdot \vec{n})\, dA$$

An approximate form of the angular momentum equation in terms of average properties at inlets and outlets:

$$\sum \vec{M} = \frac{d}{dt} \int_{CV} (\vec{r} \times \vec{V}) \rho \, dV + \sum_{\text{out}} \vec{r} \times \dot{m} \vec{V} - \sum_{\text{in}} \vec{r} \times \dot{m} \vec{V}$$

$$\text{Steady flow:} \qquad \sum \vec{M} = \sum_{\text{out}} \vec{r} \times \dot{m} \vec{V} - \sum_{\text{in}} \vec{r} \times \dot{m} \vec{V}$$

The net torque acting on the control volume during steady flow is equal to the difference between the outgoing and incoming angular momentum flow rates.

$$\sum M = \sum_{\text{out}} r \dot{m} V - \sum_{\text{in}} r \dot{m} V$$

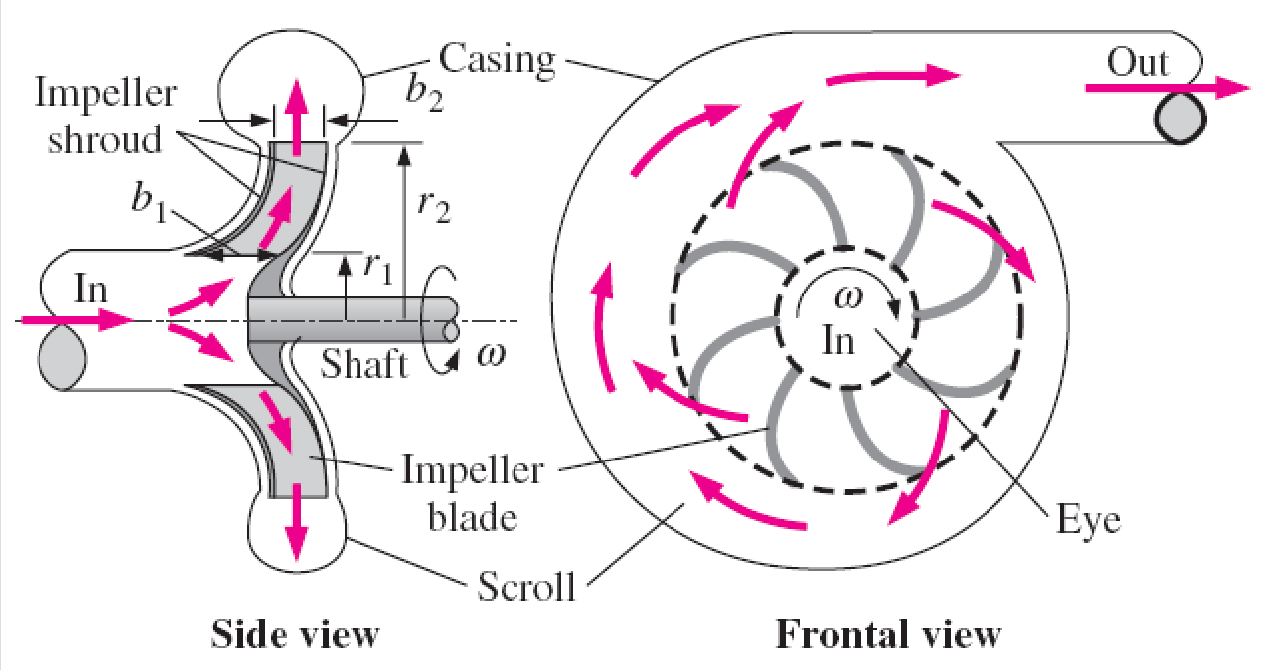
scalar form of angular momentum equation

Radial-Flow Devices

Radial-flow devices: Many rotary-flow devices such as centrifugal pumps and fans involve flow in the radial direction normal to the axis of rotation.

Axial-flow devices are easily analyzed using the linear momentum equation.

Radial-flow devices involve large changes in angular momentum of the fluid and are best analyzed with the help of the angular momentum equation.



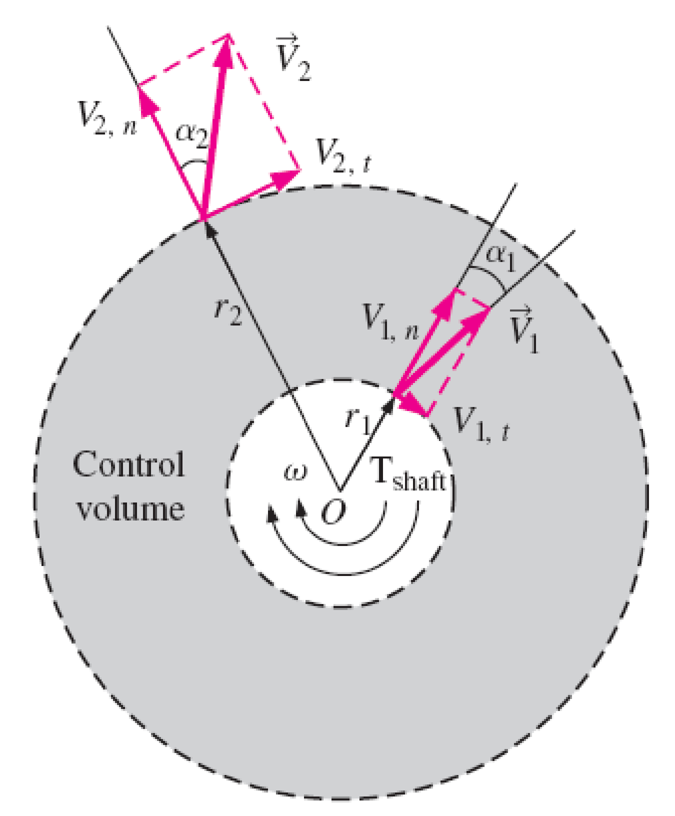
Side and frontal views of a typical centrifugal pump.

The conservation of mass equation for steady incompressible flow

$$\dot{V}_1 = \dot{V}_2 = \dot{V} \quad \rightarrow \quad (2\pi r_1 b_1)V_{1,n} = (2\pi r_2 b_2)V_{2,n}$$

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} \quad \text{and} \quad V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2}$$

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$$ angular momentum equation



$$T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t})$$ Euler's turbine formula

$$T_{shaft} = \dot{m}(r_2 V_2 \sin\alpha_2 - r_1 V_1 \sin\alpha_1)$$

When $V_{1,t} = \omega r_1$ $V_{2,t} = \omega r_2$

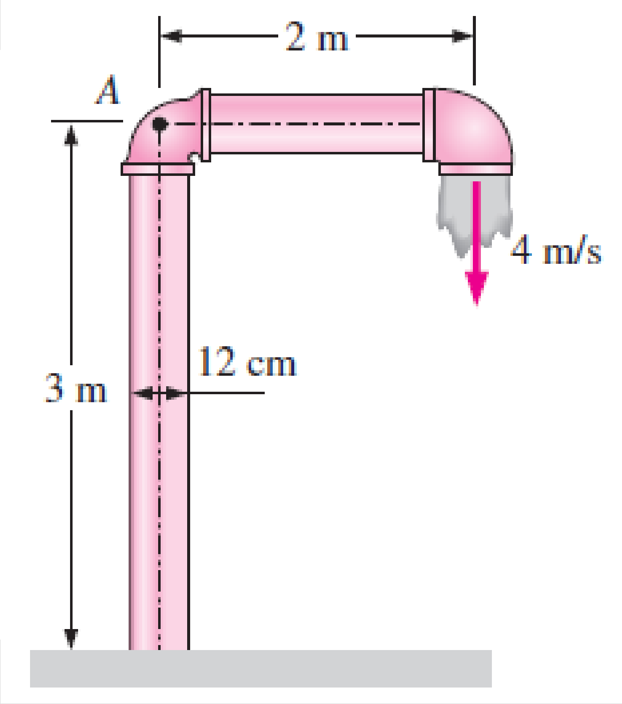
$$T_{shaft,\ ideal} = \dot{m}\omega(r_2^2 - r_1^2)$$

$$\dot{W}_{shaft} = \omega T_{shaft} = 2\pi\dot{n}T_{shaft}$$ $$\omega = 2\pi\dot{n}$$

An annular control volume that encloses the impeller section of a centrifugal pump.

Example:

Water is flowing through a 12-cm-diameter pipe that consists of a 3-m-long vertical and 2-m-long horizontal section with a 90° elbow at the exit to force the water to be discharged downward, as shown in Fig. P6–47, in the vertical direction. Water discharges to atmospheric air at a velocity of 4 m/s, and the mass of the pipe section when filled with water is 15 kg per meter length. Determine the moment acting at the intersection of the vertical and horizontal sections of the pipe (point *A*). What would your answer be if the flow were discharged upward instead of downward?



Example:

A large lawn sprinkler with four identical arms is to be converted into a turbine to generate electric power by attaching a generator to its rotating head, as shown in Fig. 6–38. Water enters the sprinkler from the base along the axis of rotation at a rate of 20 L/s and leaves the nozzles in the tangential direction. The sprinkler rotates at a rate of 300 rpm in a horizontal plane. The diameter of each jet is 1 cm, and the normal distance between the axis of rotation and the center of each nozzle is 0.6 m. Estimate the electric power produced.