COURSE OF THERMOFLUIDS II

- Profesor:

Elkin G. Flórez S.

Profesor Titular

Dr. En Ing. Mecánica, Fluidos y Aeronáutica de la Universidad Politécnica de Cataluña, Barcelona-España

M.Sc. En Ing. Química y de Procesos, Universidad Rovira i Virgili, Tarragona-España

M.Sc. En Ing. Mecánica, Universidad de los Andes, Bogotá-Colombia

Ing. Mecánico, Universidad Francisco de Paula Santander, Cúcuta-Colombia

- Oficina y Horario de Atención:

1er. Piso del Edificio Virgilio Barco (Fac. de Ingenierías y Arquitectura)

Martes de 8:00 AM a 12:00 M

Contenido Programático: Página web del programa

COURSE OF THERMOFLUIDS I

ARTÍCULO 32.- Aplicación de Evaluaciones:

a.Establézcase las semanas quinta (5), décima primera (11) y décima sexta (16), como fechas para realizar las evaluaciones de cada una de las asignaturas de un programa académico.

b. En las semanas que se realicen las evaluaciones parciales, no se desarrollarán clases de los cursos respectivos del programa académico.

PRIMER CORTE		SEGUNDO CORTE		TERCER CORTE	
35%		35%		30%	
20% Prueba escrita	15% Quices, trabajos, etc.	20% Prueba escrita	15% Quices, trabajos, etc.	20% Prueba escrita	10% Quices, trabajos, etc.

El Examen de Habilitación se presentará en la fecha y hora fijada por la Universidad, en un lapso no menor de cinco (5) días calendario, entre el examen final de un curso y su habilitación. La calificación obtenida en el Examen de Habilitación, reemplazará la nota definitiva de esta asignatura.

Fundamentals of Thermal-Fluid Sciences, 3rd Edition
Yunus A. Cengel, Robert H. Turner, John M. Cimbala
McGraw-Hill, 2008

FIRTS UNIT

MECHANISMS OF HEAT TRANSFER

Mehmet Kanoglu

Objectives

- Understand the basic mechanisms of heat transfer, which are conduction, convection, and radiation, and Fourier's law of heat conduction, Newton's law of cooling, and the Stefan–Boltzmann law of radiation
- Identify the mechanisms of heat transfer that occur simultaneously in practice
- Develop an awareness of the cost associated with heat losses
- Solve various heat transfer problems encountered in practice

INTRODUCTION

- **Heat:** The form of energy that can be transferred from one system to another as a result of temperature difference.
- **Thermodynamics** concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- **Heat Transfer** deals with the determination of the *rates* of such energy transfers as well as variation of temperature.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one.
- Heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in three different modes:

conduction*, *convection*, *radiation

CONDUCTION

Conduction: The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

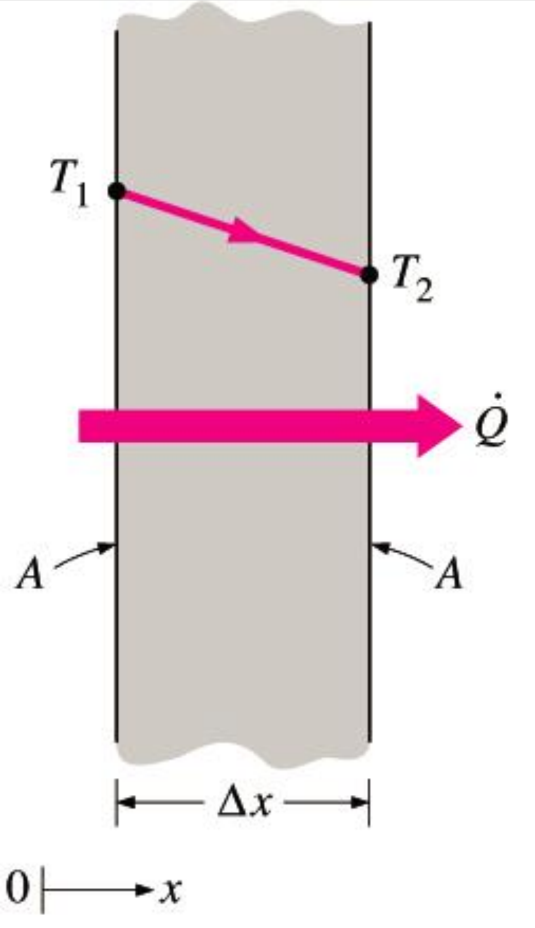
In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion.

In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons.*

The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.

$$\text{Rate of heat conduction} \propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad (\text{W})$$



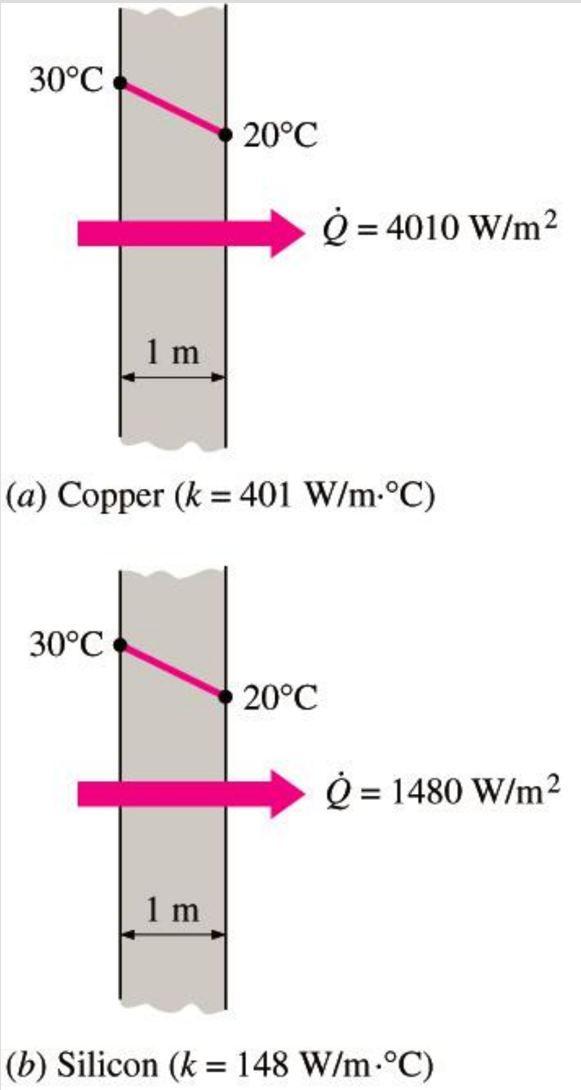
Heat conduction through a large plane wall of thickness Δx and area *A.*

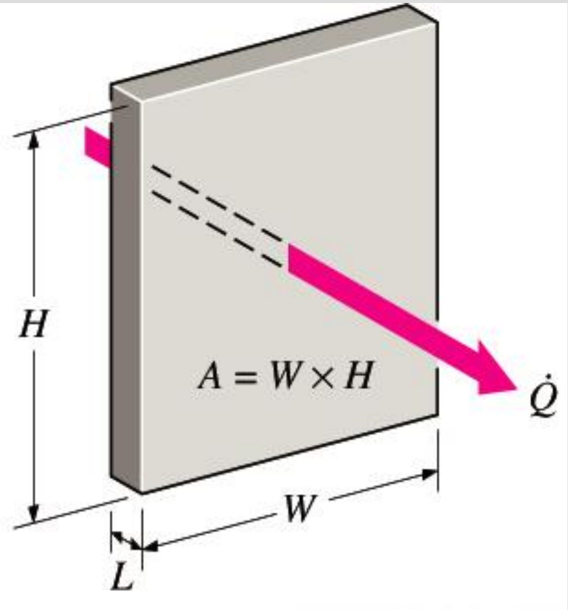
When $x \rightarrow 0$ $\dot{Q}_{cond} = -kA\frac{dT}{dx}$ **Fourier's law of heat conduction**

Thermal conductivity, *k*: A measure of the ability of a material to conduct heat.

Temperature gradient *dT/dx*: The slope of the temperature curve on a *T-x* diagram.

Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing *x*. The *negative sign* in the equation ensures that heat transfer in the positive *x* direction is a positive quantity.





In heat conduction analysis, *A* represents the area *normal* to the direction of heat transfer.

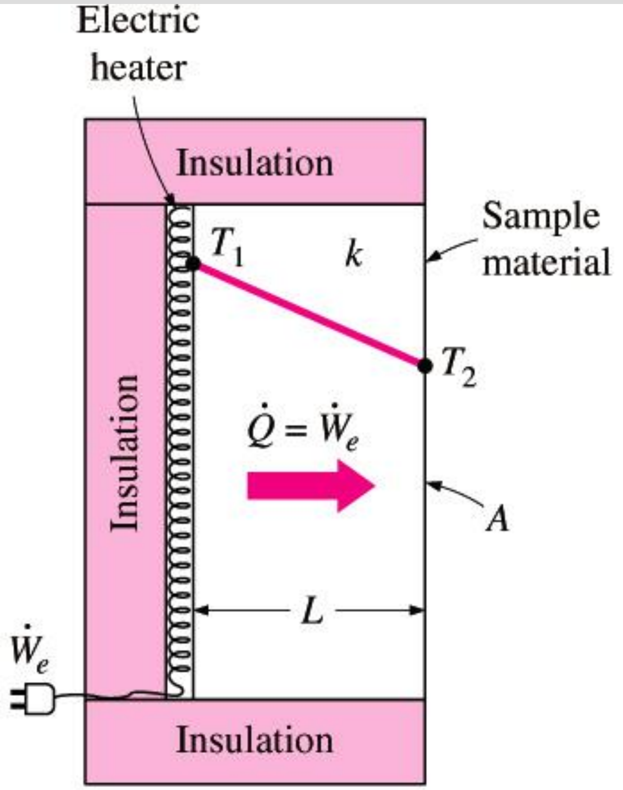
The rate of heat conduction through a solid is directly proportional to its thermal conductivity.

Thermal Conductivity

Thermal conductivity: The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.*

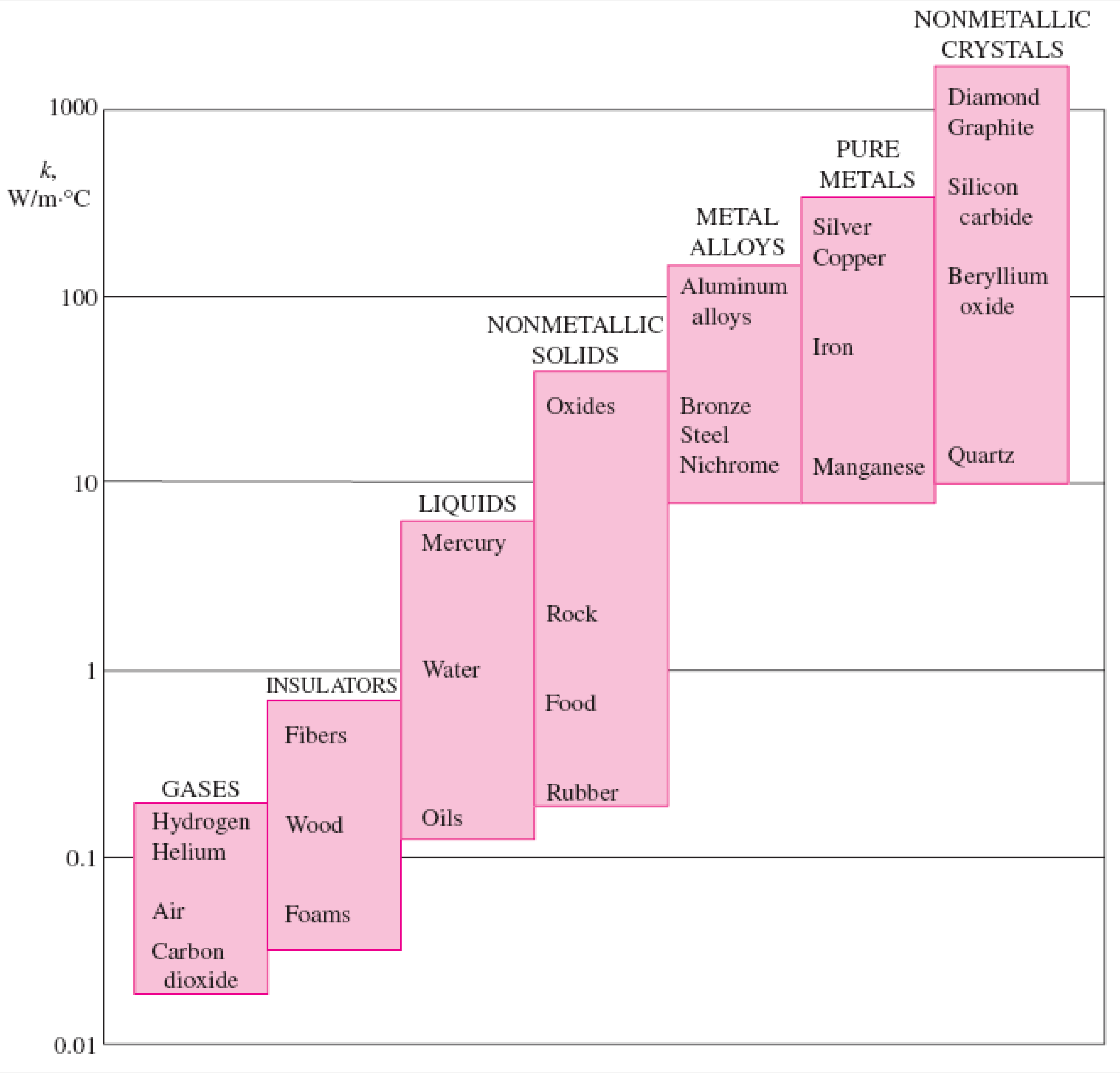


$$k = \frac{L}{A(T_1 - T_2)}\dot{Q}$$

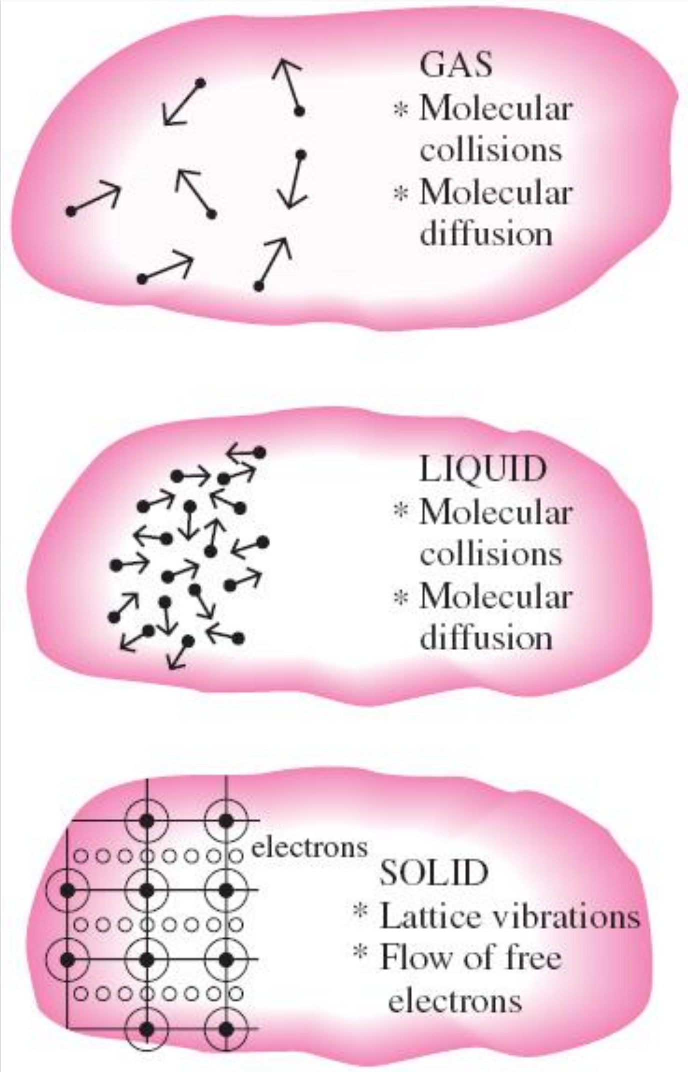
A simple experimental setup to determine the thermal conductivity of a material.

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026



The range of thermal conductivity of various materials at room temperature.



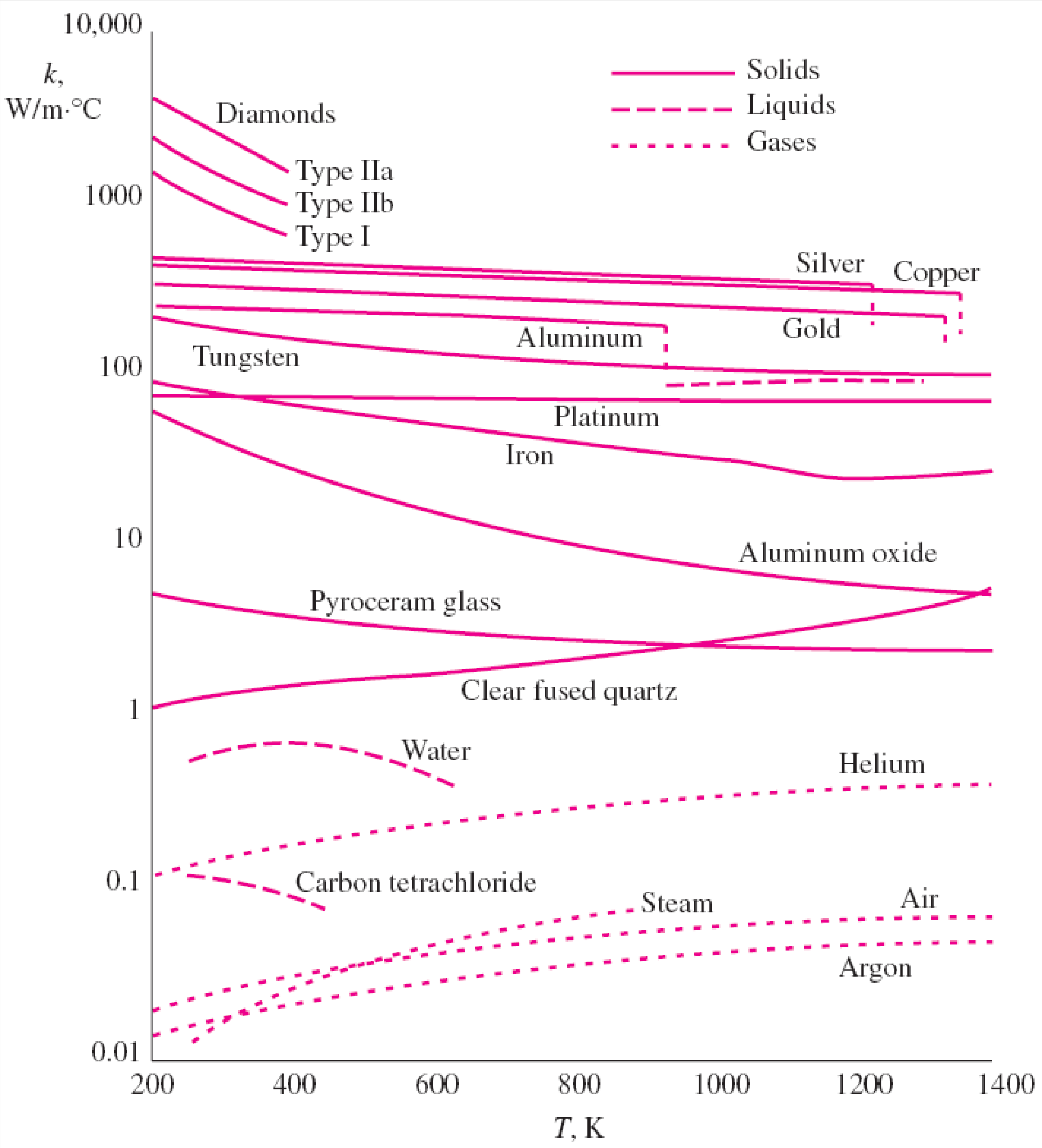
The mechanisms of heat conduction in different phases of a substance.

The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper.

Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	k, W/m · °C, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52



Thermal conductivities of materials vary with temperature

	k, W/m · °C	
T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

The variation of the thermal conductivity of various solids, liquids, and gases with temperature.

Thermal Diffusivity

c_p **Specific heat, J/kg · °C:** Heat capacity per unit mass

ρc_p **Heat capacity, J/m³ · °C:** Heat capacity per unit volume

α **Thermal diffusivity, m²/s:** Represents how fast heat diffuses through a material

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho c_p} \qquad (\text{m}^2/\text{s})$$

A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.

The larger the thermal diffusivity, the faster the propagation of heat into the medium.

A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

The thermal diffusivities of some materials at room temperature

Material	α, m²/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

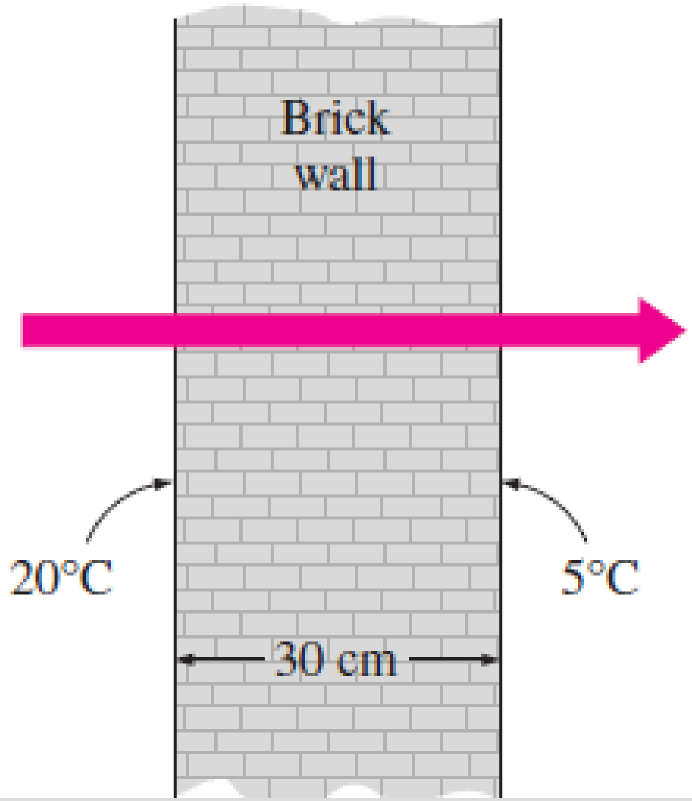
Quiz:

Define

- Thermal Diffusivity
- Thermal Conductivity

Exercises:

The inner and outer surfaces of a 5-m × 6-m brick wall of thickness 30 cm and thermal conductivity 0.69 W/m · °C are maintained at temperatures of 20°C and 5°C, respectively. Determine the rate of heat transfer through the wall, in W.

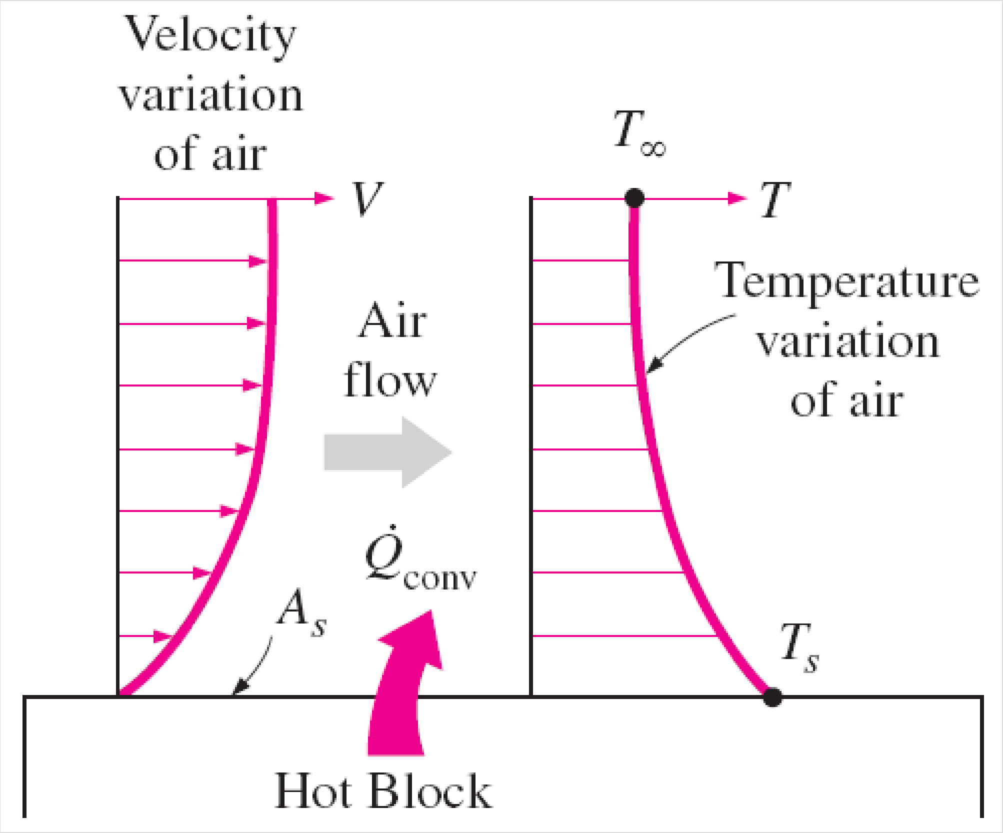


CONVECTION

Convection: The mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion*.

The faster the fluid motion, the greater the convection heat transfer.

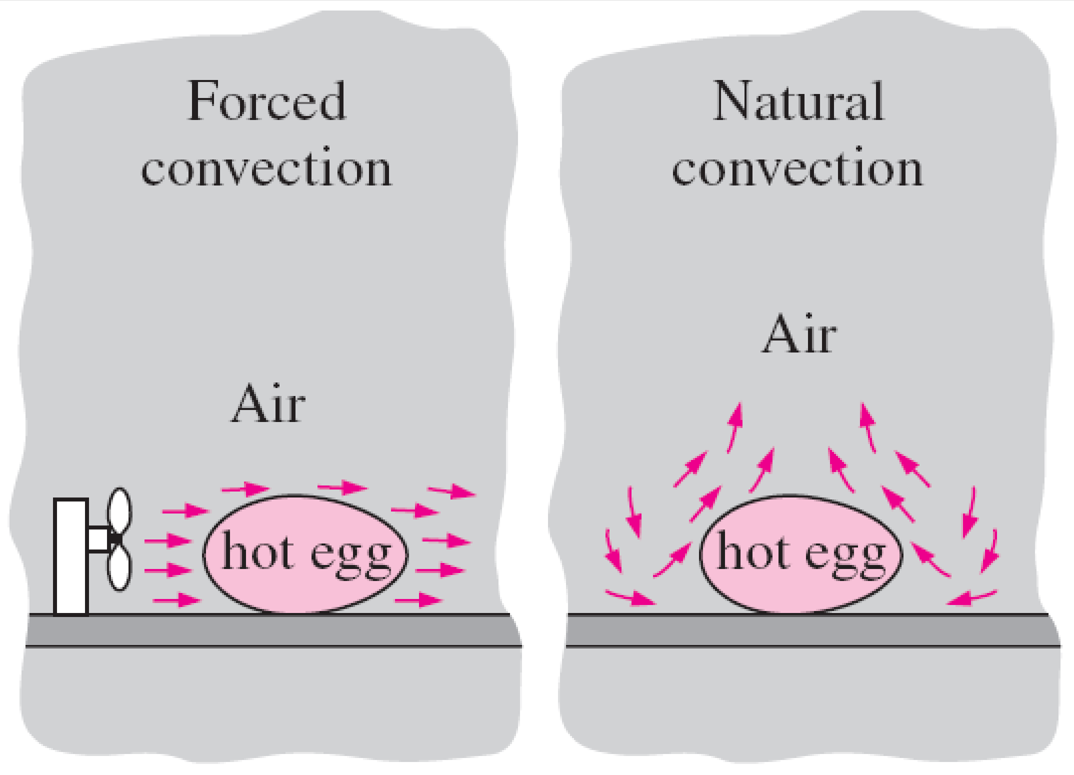
In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.



Heat transfer from a hot surface to air by convection.

Forced convection: If the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.

Natural (or **free**) **convection:** If the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.



The cooling of a boiled egg by forced and natural convection.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (W)$$

Newton's law of cooling

h	convection heat transfer coefficient, $W/m^2 \cdot °C$
A_s	the surface area through which convection heat transfer takes place
T_s	the surface temperature
T_∞	the temperature of the fluid sufficiently far from the surface.

The convection heat transfer coefficient *h* is not a property of the fluid.

It is an experimentally determined parameter whose value depends on all the variables influencing convection such as

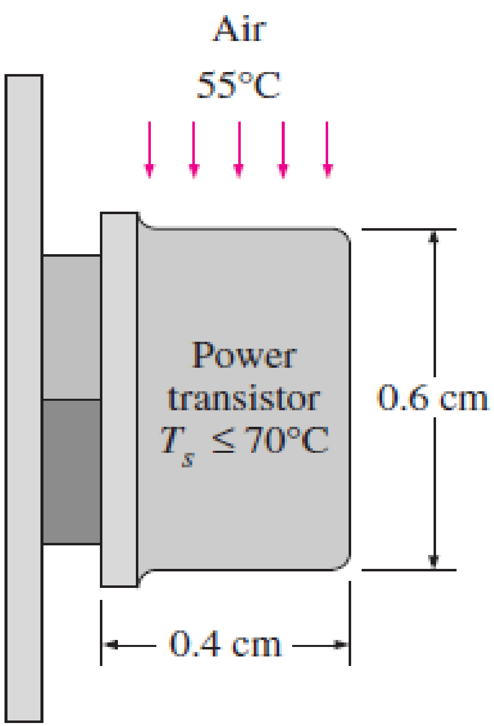
- the surface geometry
- the nature of fluid motion
- the properties of the fluid
- the bulk fluid velocity

Typical values of convection heat transfer coefficient

Type of convection	h, $W/m^2 \cdot °C$*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

Exercises:

A transistor with a height of 0.4 cm and a diameter of 0.6 cm is mounted on a circuit board. The transistor is cooled by air flowing over it with an average heat transfer coefficient of 30 W/m^2 · °C. If the air temperature is 55°C and the transistor case temperature is not to exceed 70°C, determine the amount of power this transistor can dissipate safely. Disregard any heat transfer from the transistor base.



RADIATION

- **Radiation:** The energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium.*
- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
- However, radiation is usually considered to be a *surface phenomenon* for solids.

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \quad \text{(W)}$$

Stefan–Boltzmann law

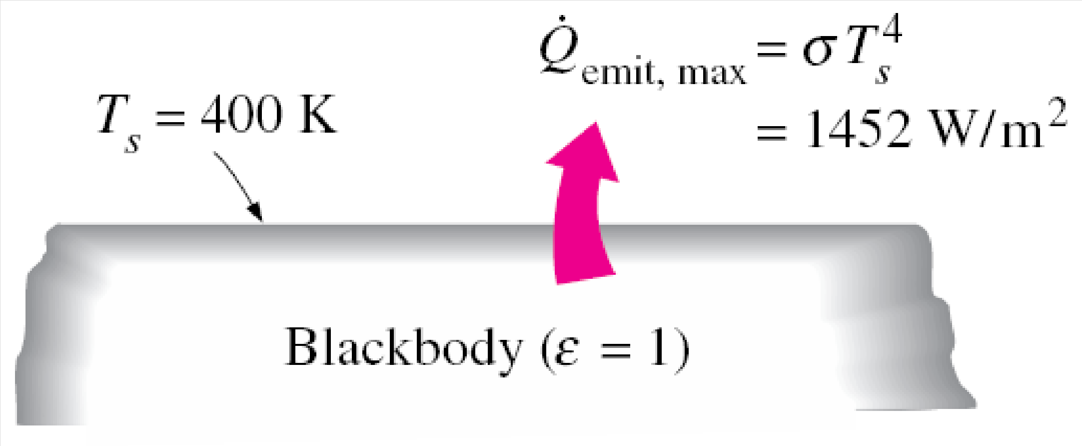
$\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ *Stefan–Boltzmann constant*

Blackbody: The idealized surface that emits radiation at the maximum rate.

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \quad \text{(W)}$$

Radiation emitted by real surfaces

Emissivity ε : A measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$ of the surface. **$0 \leq \varepsilon \leq 1$**.



Blackbody radiation represents the *maximum amount of radiation that can be emitted from a surface at a specified temperature.*

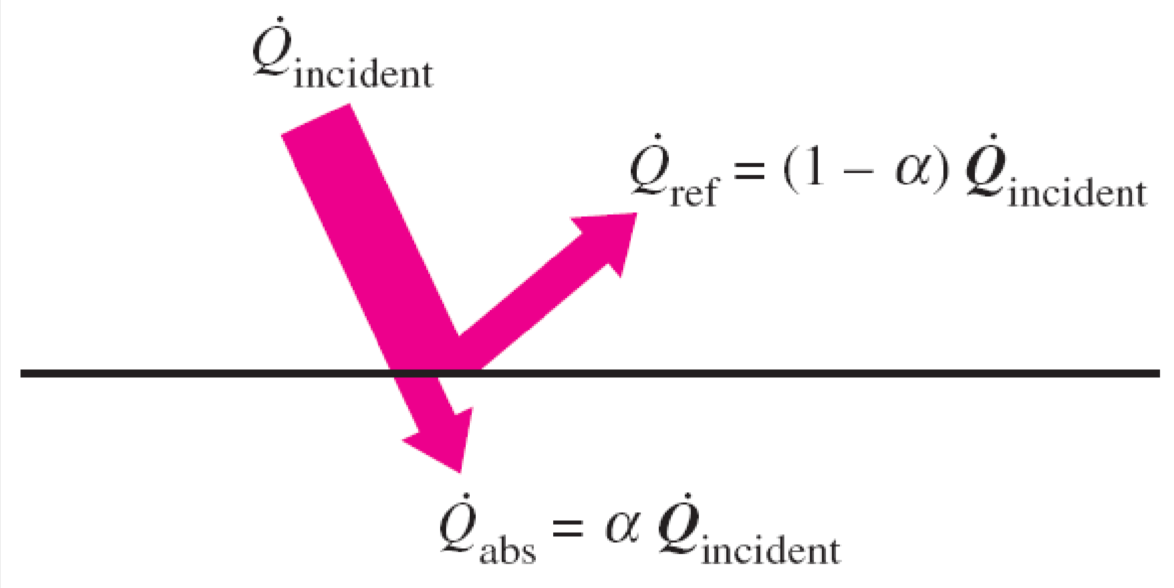
Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

Absorptivity α: The fraction of the radiation energy incident on a surface that is absorbed by the surface. **$0 \leq \alpha \leq 1$**

A blackbody absorbs the entire radiation incident on it (**$\alpha = 1$**).

Kirchhoff's law: The emissivity and the absorptivity of a surface at a given temperature and wavelength are equal.



The absorption of radiation incident on an opaque surface of absorptivity .

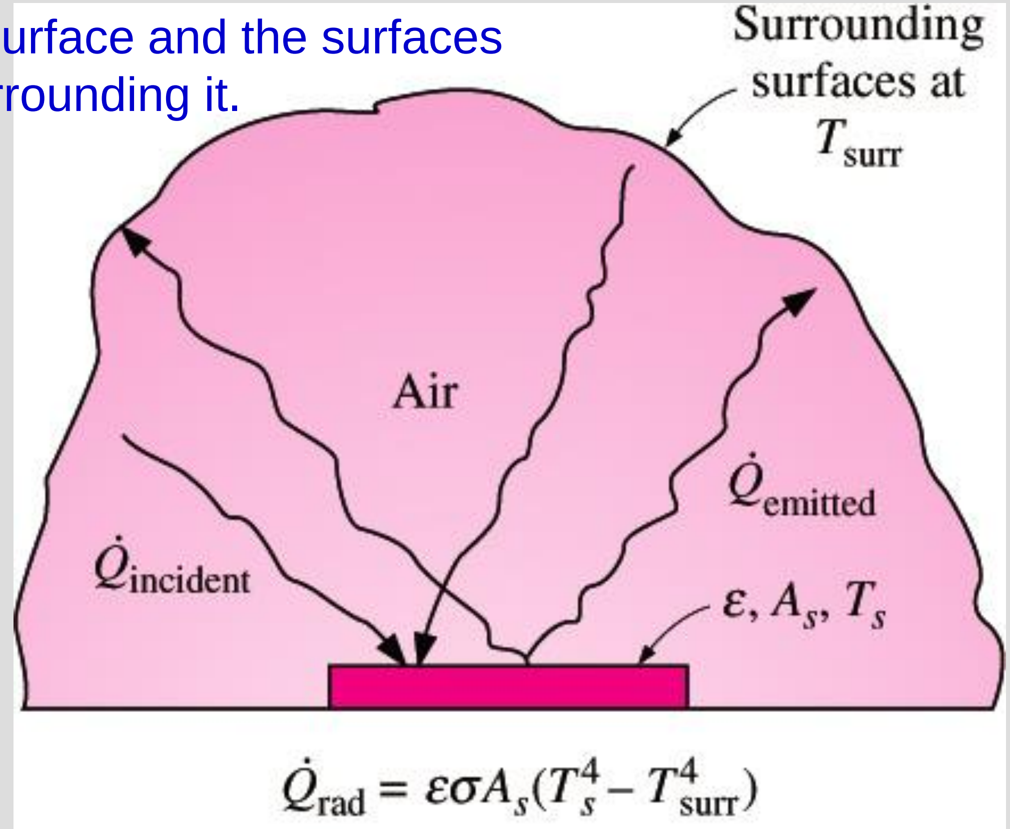
Net radiation heat transfer: The difference between the rates of radiation emitted by the surface and the radiation absorbed.

The determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on

- the properties of the surfaces
- their orientation relative to each other
- the interaction of the medium between the surfaces with radiation

Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection.

Radiation heat transfer between a surface and the surfaces surrounding it.



When radiation and convection occur simultaneously between a surface and a gas

$$\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty) \qquad \text{(W)}$$

Combined heat transfer coefficient $h_{combined}$
Includes the effects of both convection and radiation

SIMULTANEOUS HEAT TRANSFER MECHANISMS

Heat transfer is only by conduction in *opaque solids,* but by conduction and radiation in *semitransparent solids.*

A solid may involve conduction and radiation but not convection. A solid may involve convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid.*

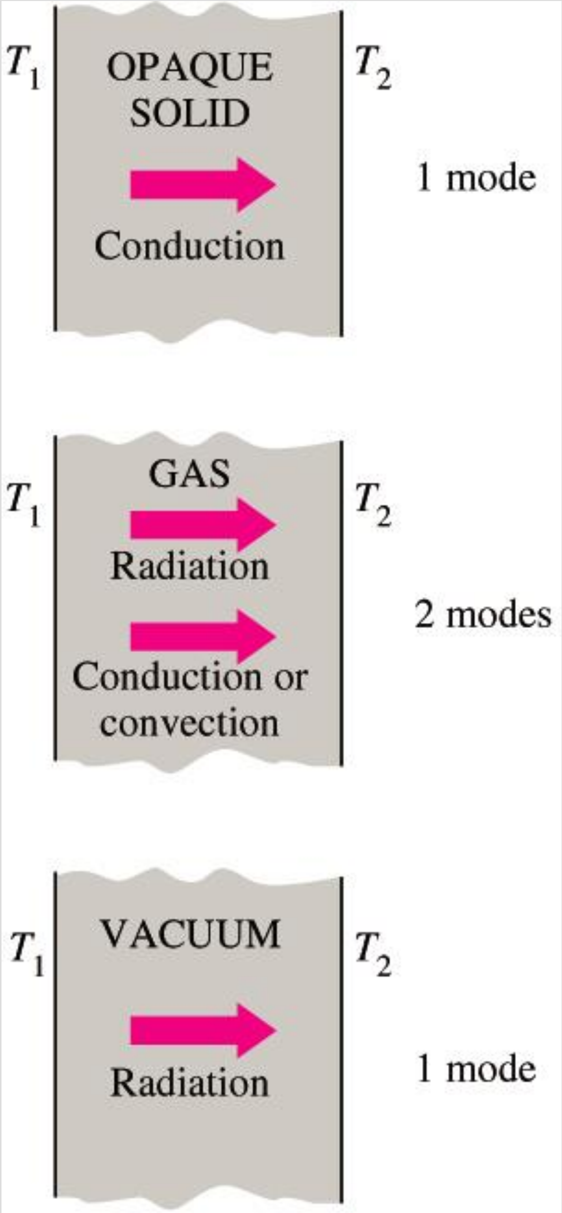
In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.

Convection = Conduction + Fluid motion

Heat transfer through a *vacuum* is by radiation.

Most gases between two solid surfaces do not interfere with radiation.

Liquids are usually strong absorbers of radiation.



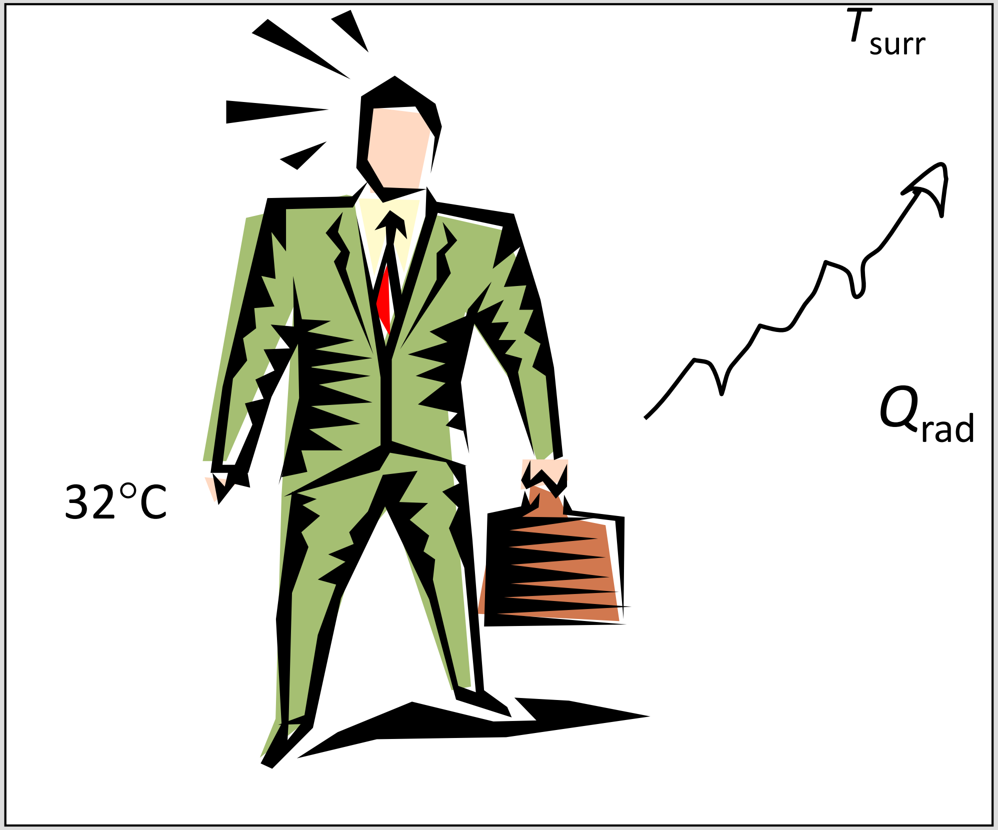
Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

Summary

- Conduction
  - ✓ Fourier’s law of heat conduction
  - ✓ Thermal Conductivity
  - ✓ Thermal Diffusivity
- Convection
  - ✓ Newton’s law of cooling
- Radiation
  - ✓ Stefan–Boltzmann law
- Simultaneous Heat Transfer Mechanisms

Exercises:

Consider a person whose exposed surface area is 1.7 m^2, emissivity is 0.7, and surface temperature is 32°C. Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of *a*) 300 K and *b*) 280 K.



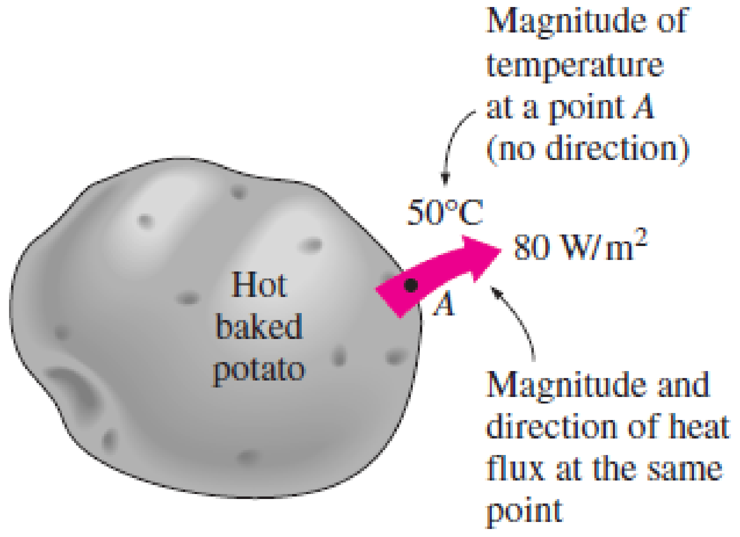
Fundamentals of Thermal-Fluid Sciences, 3rd Edition
Yunus A. Cengel, Robert H. Turner, John M. Cimbala
McGraw-Hill, 2008

HEAT CONDUCTION EQUATION

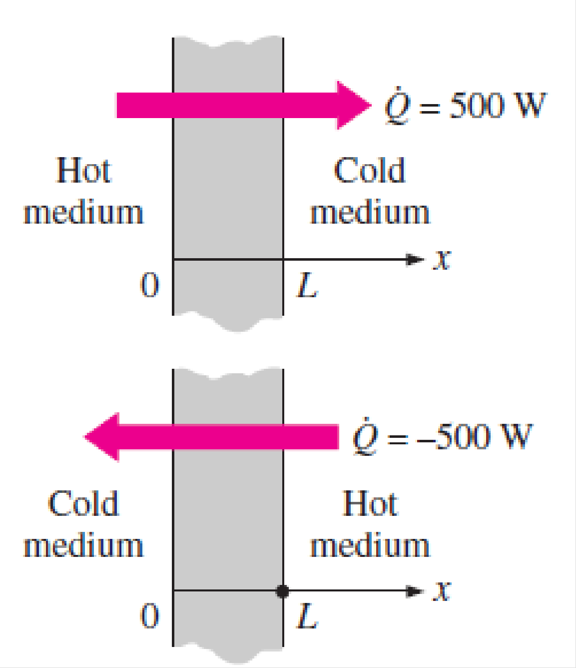
Mehmet Kanoglu

Introduction

- Unlike temperature, heat transfer has direction as well as magnitude, and thus it is a *vector* quantity (see figure).



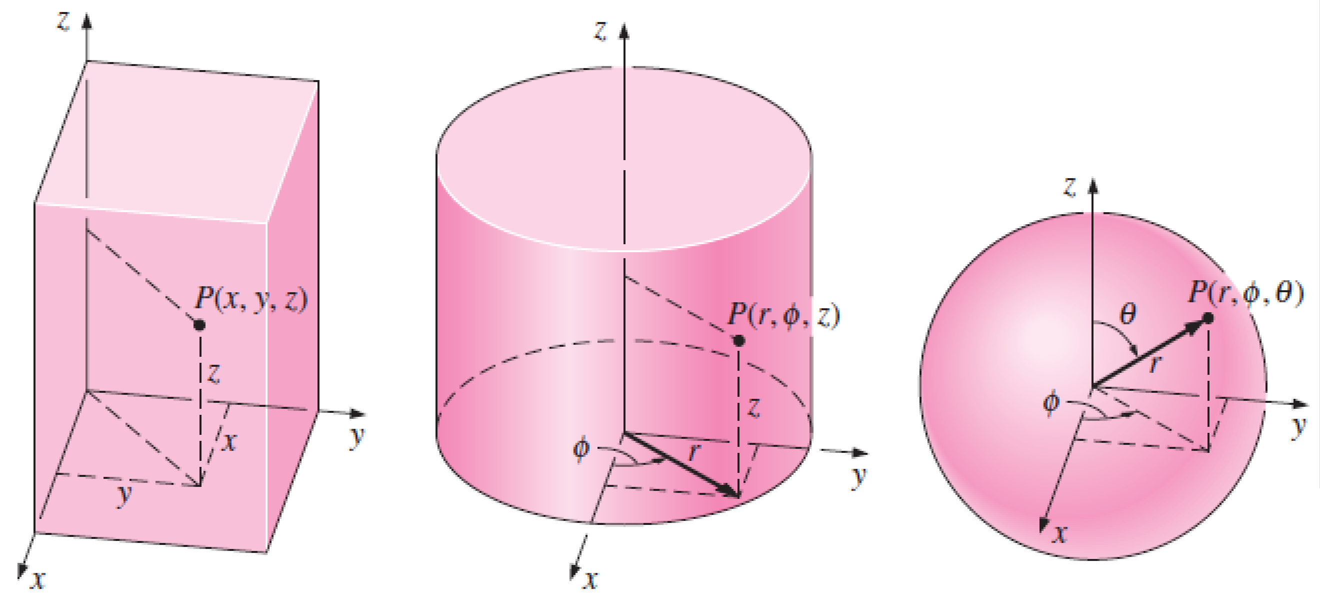
Heat transfer has direction as well as magnitude, and thus it is a *vector* quantity.



To avoid such questions, we can work with a coordinate system and indicate direction with plus or minus signs. The generally accepted convention is that heat transfer in the positive direction of a coordinate axis is positive and in the opposite direction it is negative. Therefore, a positive quantity indicates heat transfer in the positive direction and negative quantity indicate heat transfer in the negative direction.

Introduction

- The specification of the temperature at a point in a medium first requires the specification of the location of that point. This can be done by choosing a suitable coordinate system such as the rectangular, cylindrical, or spherical coordinates, depending on the geometry involved, and a convenient reference point (the origin).

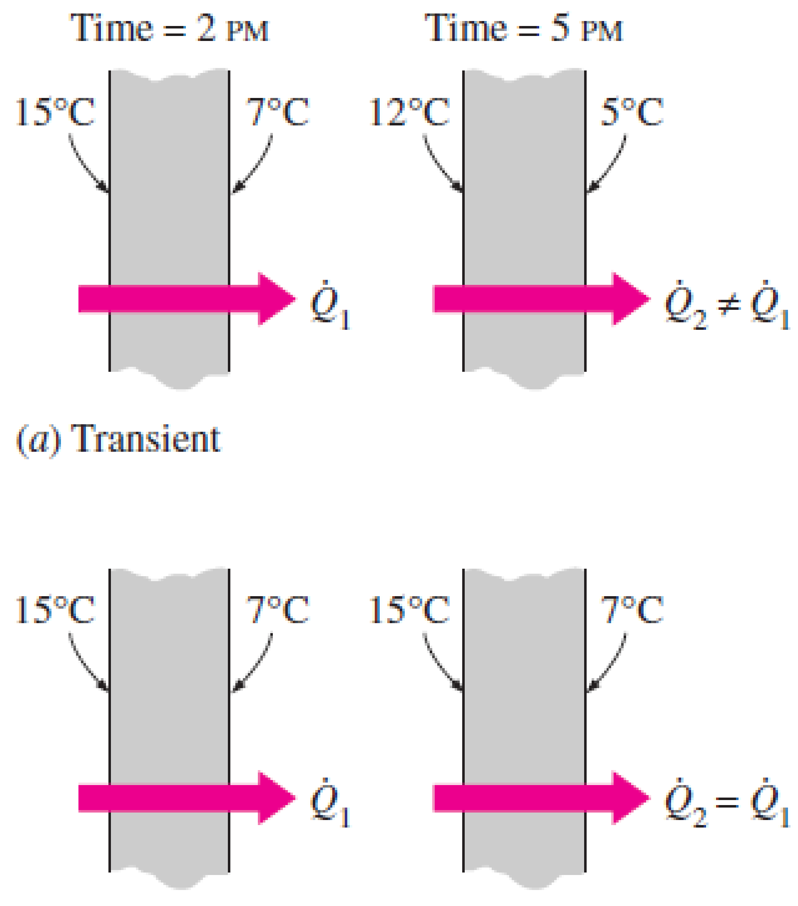


(a) Rectangular coordinates (b) Cylindrical coordinates (c) Spherical coordinates

The various distances and angles involved when describing the location of a point in different coordinate systems.

Steady versus Transient Heat Transfer

- Heat transfer problems are often classified as being **steady** (also called steady-state) or **transient** (also called unsteady). Te term steady implies no change with time at any point within the medium, while transient implies variation with time o time dependence. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location, although both quantities may vary from one location to another.

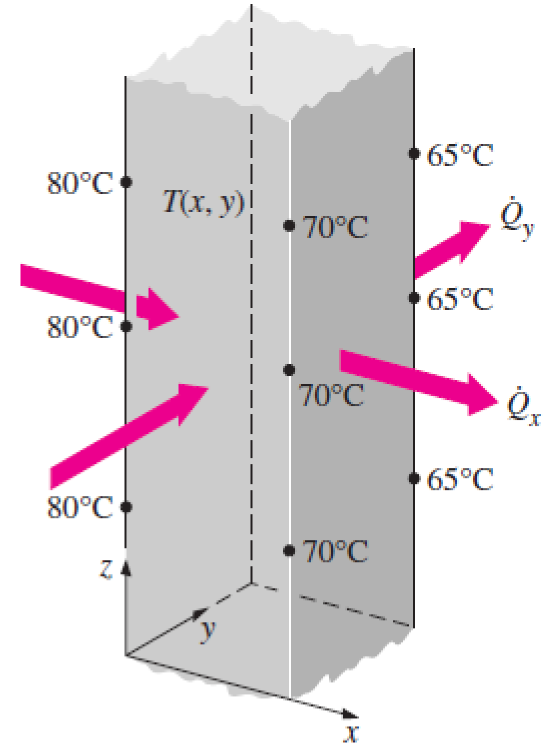


(*a*) Transient

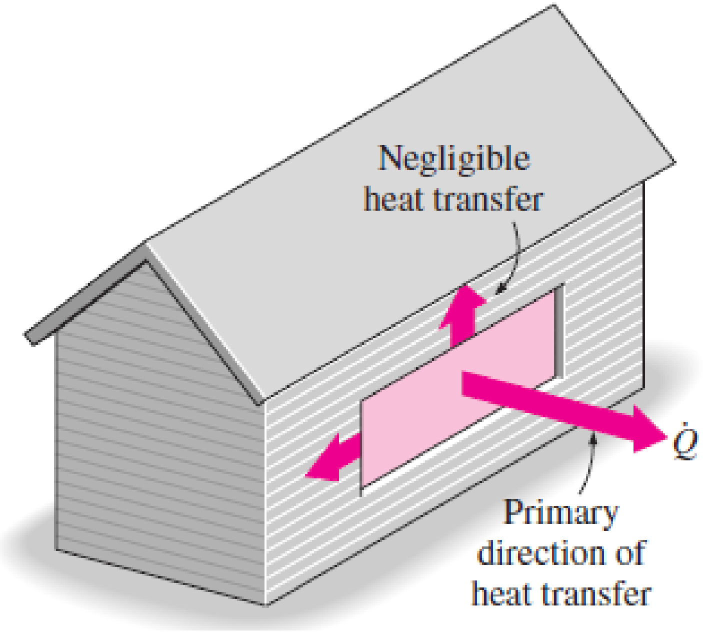
(*b*) Steady-state

Multidimensional Heat Transfer

Heat transfer problems are also classified as being *one-dimensional*, *two-dimensional*, or *three-dimensional*, depending on the relative magnitudes of heat transfer rates in different directions and the level of accuracy desired. In the most general case, heat transfer through a medium is **three-dimensional.**



Two-dimensional heat transfer in a long rectangular bar.



Heat transfer through the window of a house can be taken to be one-dimensional.

Multidimensional Heat Transfer

To obtain a general relation for Fourier's law of heat conduction, consider a medium in which the temperature distribution is three-dimensional. Figure 2–8 shows an isothermal surface in that medium. The heat flux vector at a point *P* on this surface must be perpendicular to the surface, and it must point in the direction of decreasing temperature. If *n* is the normal of the isothermal surface at point *P*, the rate of heat conduction at that point can be expressed by Fourier's law as

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad \text{(W)}$$

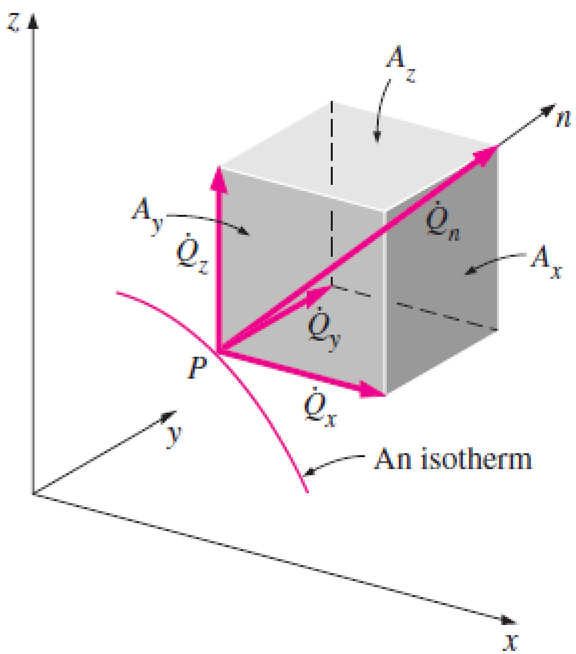
In rectangular coordinates, the heat conduction vector can be expressed in terms of its components as

$$\vec{\dot{Q}}_n = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the unit vectors, and $\dot{Q}_x$, $\dot{Q}_y$, and $\dot{Q}_z$ are the magnitudes of the heat transfer rates in the *x*-, *y*-, and *z*-directions, which again can be determined from Fourier's law as

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x}, \qquad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y}, \qquad \text{and} \qquad \dot{Q}_z = -kA_z\frac{\partial T}{\partial z}$$

Here A_x, A_y and A_z are heat conduction areas normal to the *x*-, *y*-, and *z*-directions, respectively



The heat transfer vector is always normal to an isothermal surface and can be resolved into its components like any other vector.

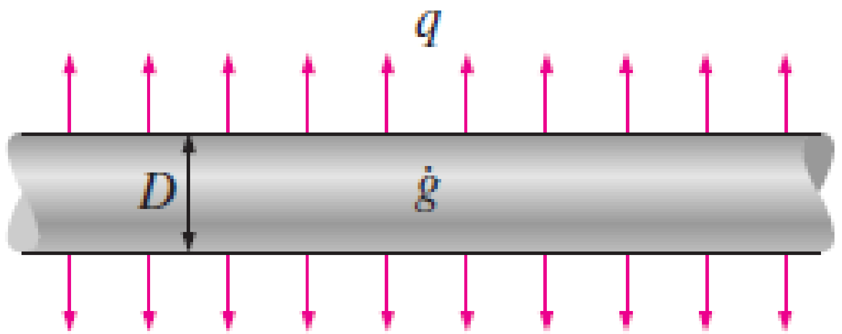
Heat Generation

heat generation is a *volumetric phenomenon*. That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified *per unit volume* and is denoted by $\dot{g}$, whose unit is W/m^3 or $Btu/h \cdot ft^3$.

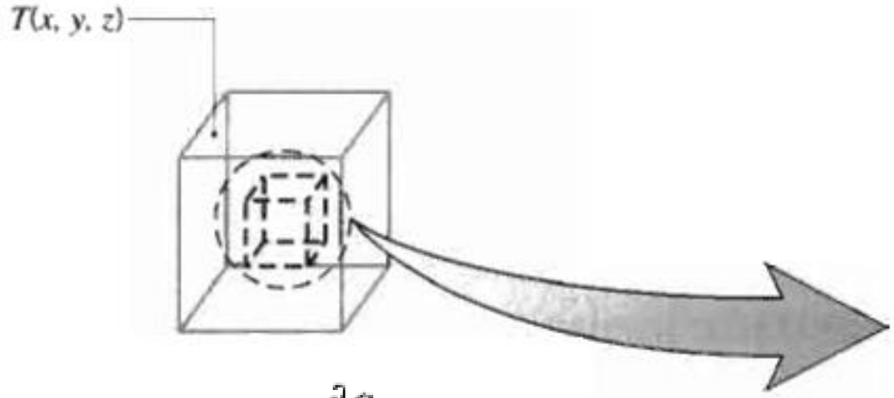
The rate of heat generation in a medium may vary with time as well as position within the medium. When the variation of heat generation with position is known, the *total* rate of heat generation in a medium of volume V can be determined from

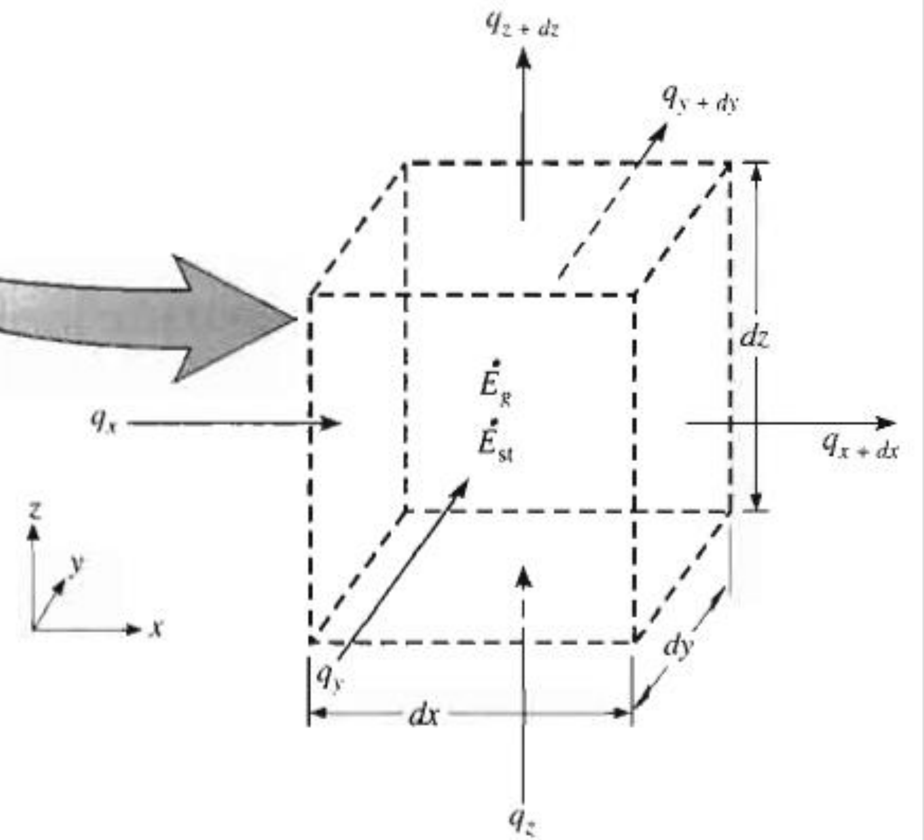
$$\dot{G} = \int_V \dot{g}\, dV \qquad \text{(W)}$$

In the special case of *uniform* heat generation, as in the case of electric resistance heating throughout a homogeneous material, the relation in Eq. 2–5 reduces to $\dot{G} = \dot{g}V$, where $\dot{g}$ is the constant rate of heat generation per unit volume.



Heat Conduction equation in Cartesian coordinates





$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\dot{E}_g = \dot{q}\, dx\, dy\, dz$$

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

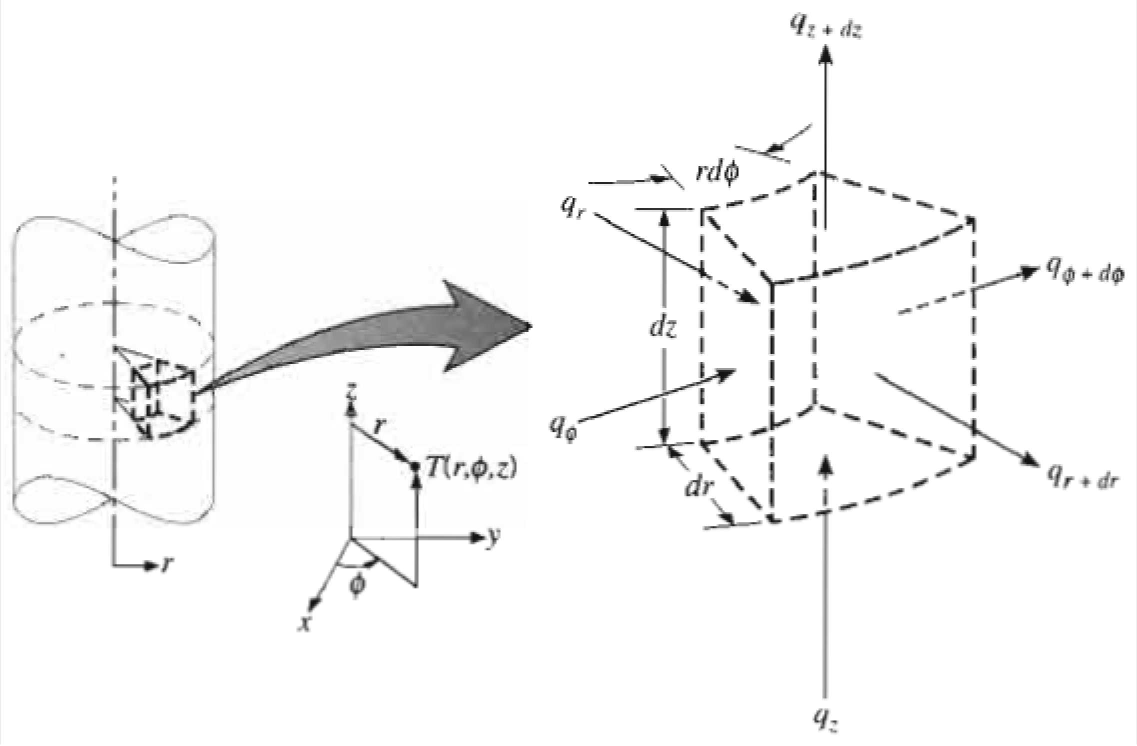
Heat Conduction equation

- The general form, in ***Cartesian coordinates*** of the heat diffusion equation is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

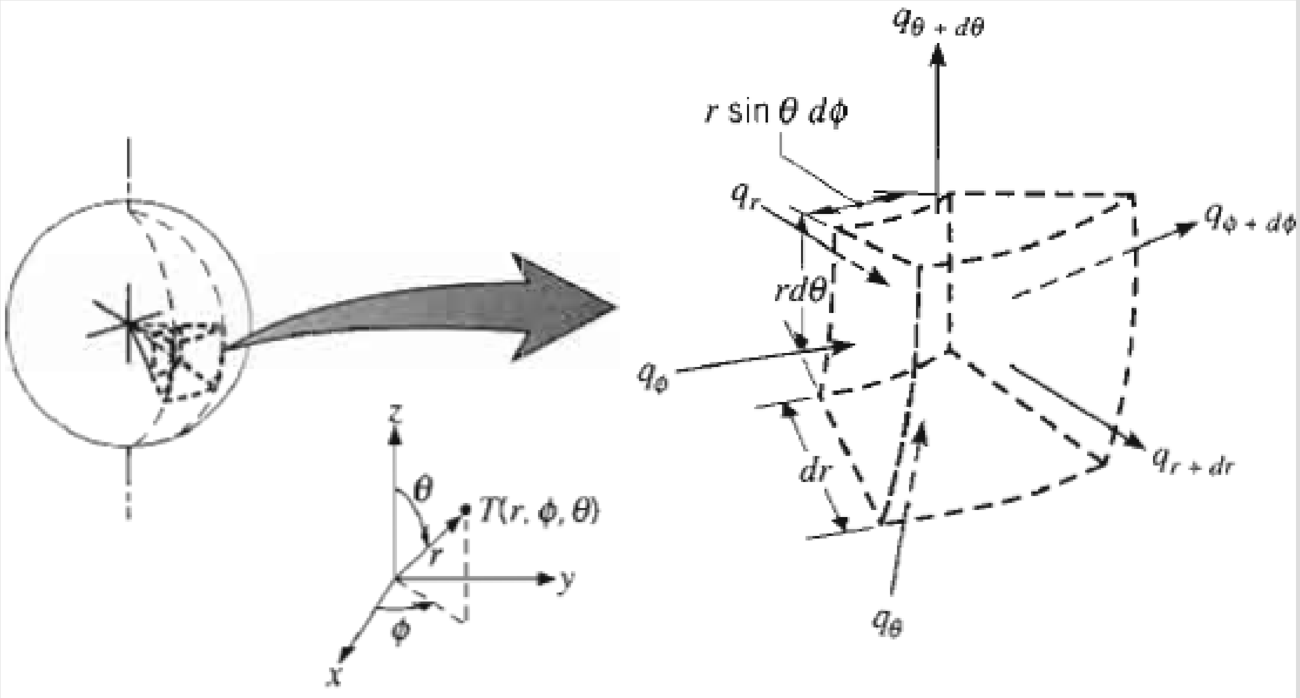
- This equation, often referred to as the *heat equation*, provides the basic tool for heat conduction analysis. From its solution, we can obtain the temperature distribution $T(x, y, z)$.

Heat Conduction equation in cylindrical coordinates



$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Heat Conduction equation in spherical coordinates



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

One-Dimensional Heat Conduction equation.

- ***Large plane wall***

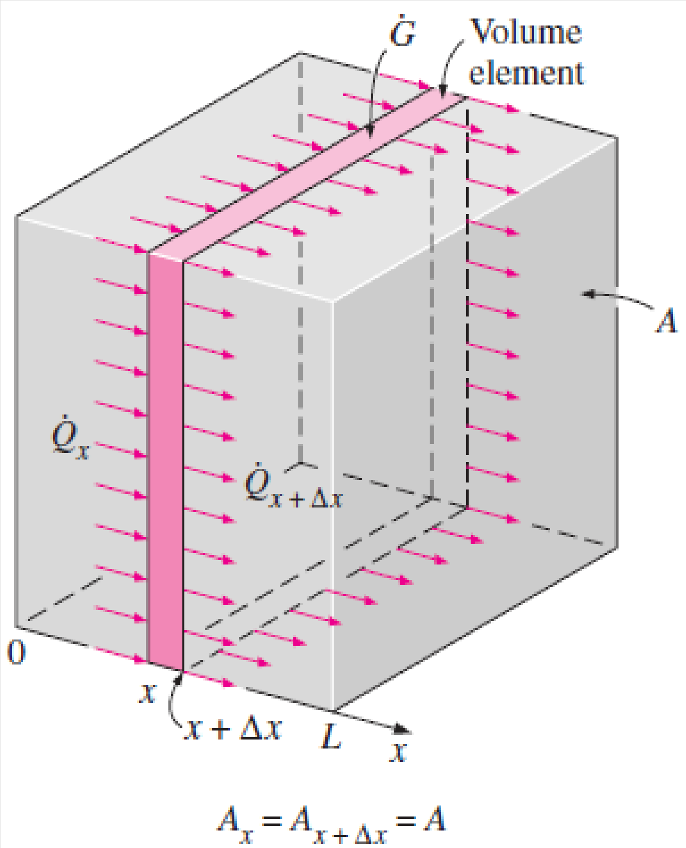
Variable conductivity: $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$

Constant conductivity: $\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(1) *Steady-state:* $(\partial/\partial t = 0)$ $\quad \frac{d^2T}{dx^2} + \frac{\dot{g}}{k} = 0$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$ $\quad \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0 \text{ and } \dot{g} = 0)$ $\quad \frac{d^2T}{dx^2} = 0$

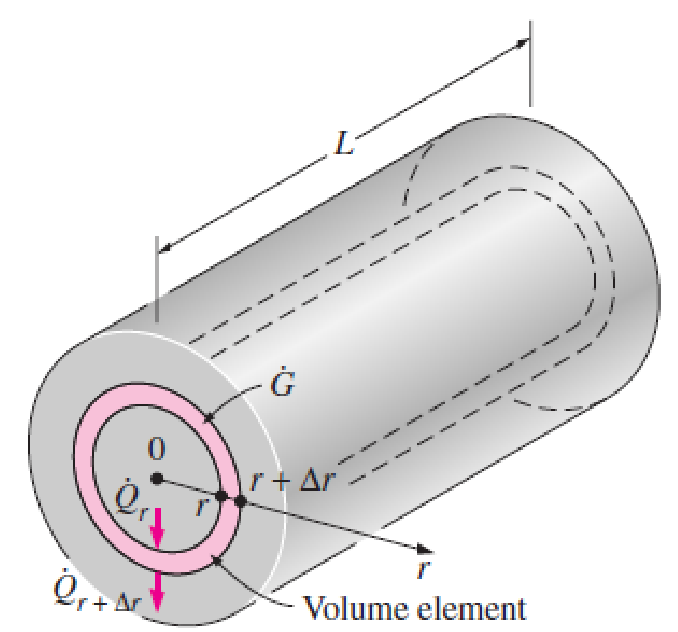


One-Dimensional Heat Conduction equation.

- ***Long cylinder***

Variable conductivity: $\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$

Constant conductivity: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$



(1) *Steady-state:* $(\partial/\partial t = 0)$ $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$ $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

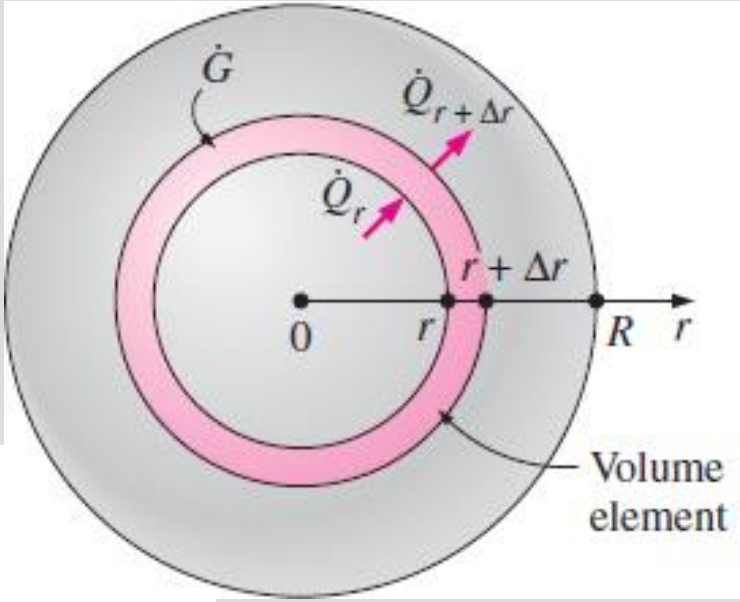
(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{g} = 0)$ $\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

One-Dimensional Heat Conduction equation.

- ***Sphere***

Variable conductivity: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$

Constant conductivity: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$



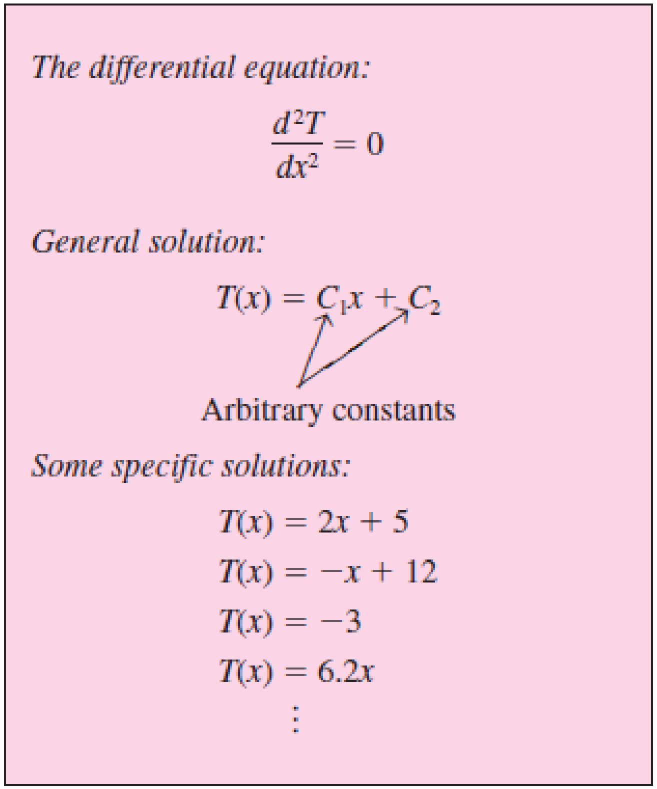
(1) *Steady-state:* $(\partial/\partial t = 0)$ $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$ $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

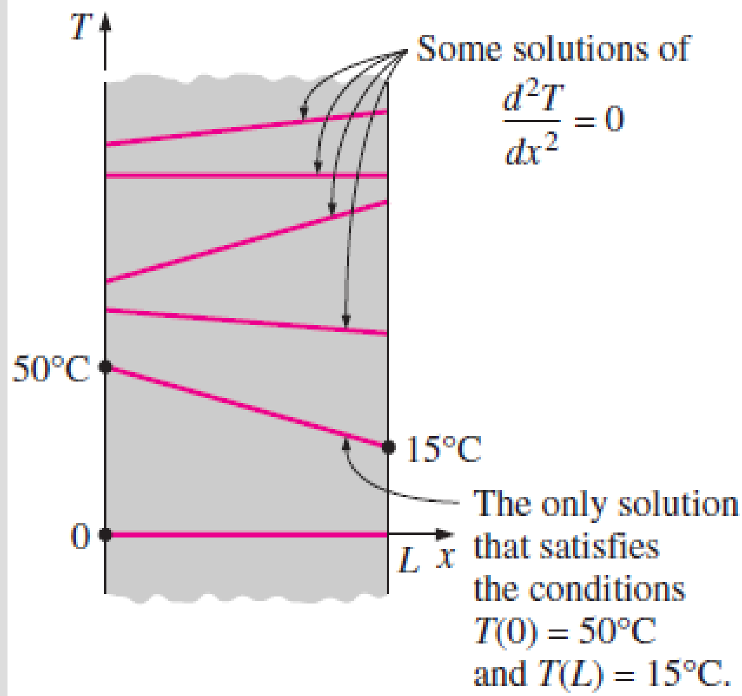
(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{g} = 0)$ $\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$ or $r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0$

One-Dimensional Heat Conduction equation.

- ***Boundary and initial conditions***



The general solution of a typical differential equation involves arbitrary constants, and thus an infinite number of solutions.



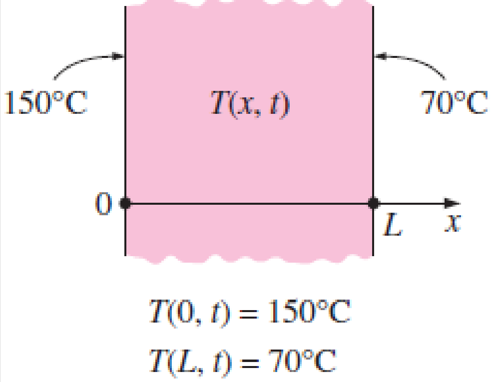
To describe a heat transfer problem completely, two boundary conditions must be given for each direction along which heat transfer is significant.

One-Dimensional Heat Conduction equation.

- ***Boundary and initial conditions***

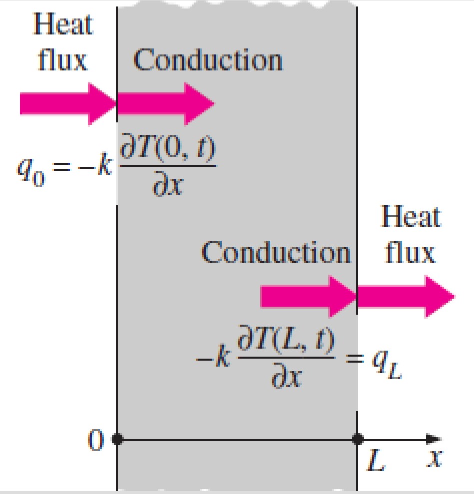
1 Specified Temperature Boundary Condition

$$T(0, t) = T_1$$
$$T(L, t) = T_2$$



2 Specified Heat Flux Boundary Condition

$$\dot{q} = -k\frac{\partial T}{\partial x} = \left(\begin{array}{c}\text{Heat flux in the}\\ \text{positive } x\text{-direction}\end{array}\right) \qquad (\text{W/m}^2)$$



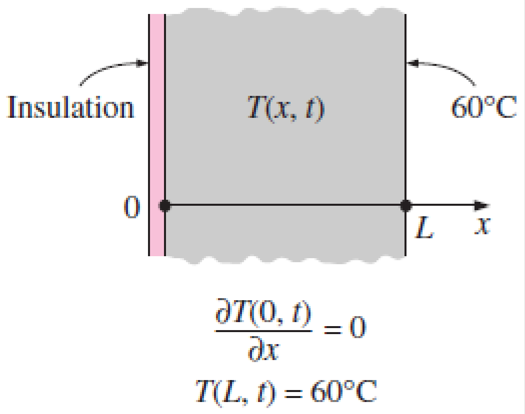
One-Dimensional Heat Conduction equation.

- ***Boundary and initial conditions***

2 Specified Heat Flux Boundary Condition

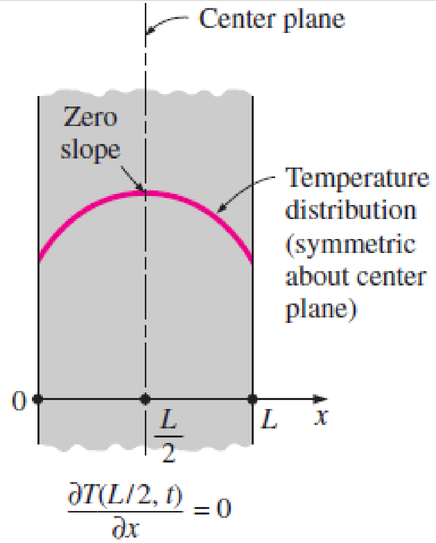
Special Case: Insulated Boundary

$$k\frac{\partial T(0,t)}{\partial x} = 0 \qquad \text{or} \qquad \frac{\partial T(0,t)}{\partial x} = 0$$



Another Special Case: Thermal Symmetry

$$\frac{\partial T(L/2,t)}{\partial x} = 0$$



One-Dimensional Heat Conduction equation.

- ***Boundary and initial conditions***

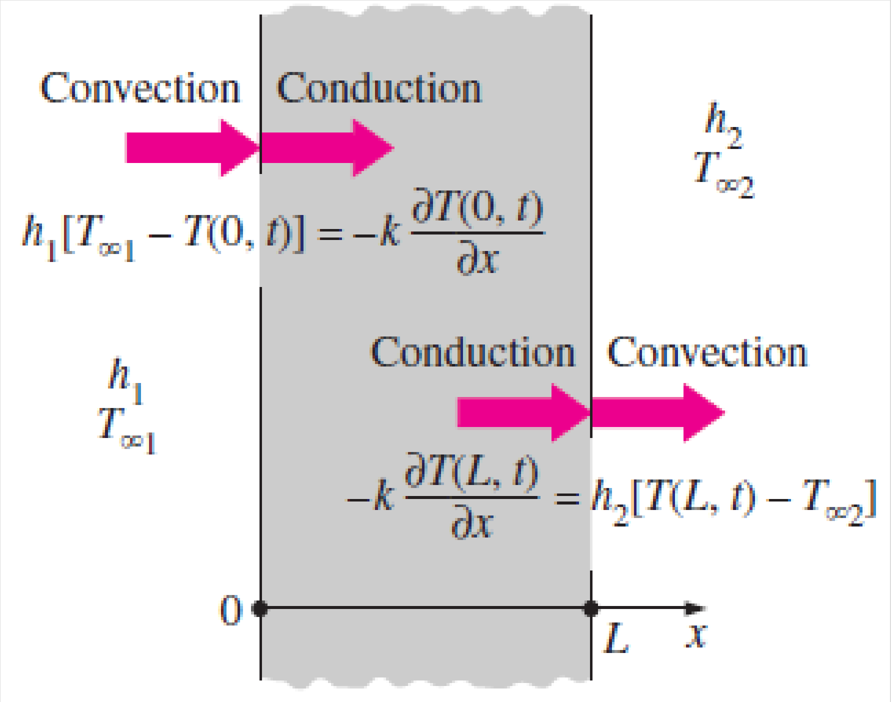
3 Convection Boundary Condition

(Heat conduction at the surface in a selected direction) = (Heat convection at the surface in the same direction)

$$-k\frac{\partial T(0,t)}{\partial x} = h_1[T_{\infty 1} - T(0,t)]$$

and

$$-k\frac{\partial T(L,t)}{\partial x} = h_2[T(L,t) - T_{\infty 2}]$$



One-Dimensional Heat Conduction equation.

- ***Boundary and initial conditions***

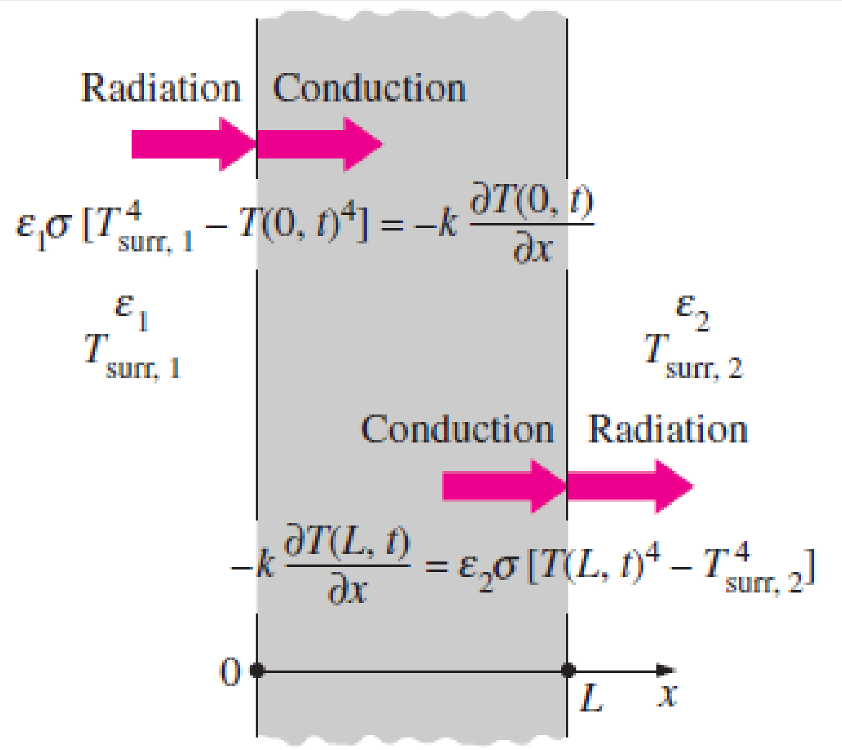
4 Radiation Boundary Condition

$$\begin{pmatrix} \text{Heat conduction} \\ \text{at the surface in a} \\ \text{selected direction} \end{pmatrix} = \begin{pmatrix} \text{Radiation exchange} \\ \text{at the surface in} \\ \text{the same direction} \end{pmatrix}$$

$$-k\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma[T_{surr,1}^4 - T(0,t)^4]$$

and

$$-k\frac{\partial T(L,t)}{\partial x} = \varepsilon_2\sigma[T(L,t)^4 - T_{surr,2}^4]$$

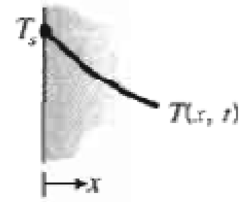


One-Dimensional Heat Conduction equation.

Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

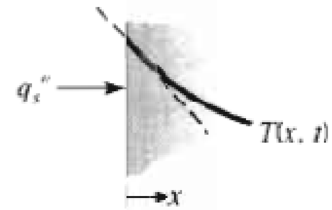
$$T(0, t) = T_s \tag{2.29}$$



2. Constant surface heat flux

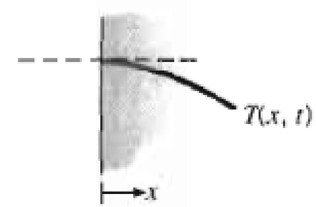
(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \tag{2.30}$$



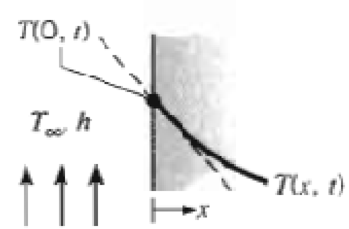
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \tag{2.31}$$



3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \tag{2.32}$$



Example:

2–56 Consider a large plane wall of thickness $L = 0.4$ m, thermal conductivity $k = 2.3$ W/m · °C, and surface area $A = 20$ m^2. The left side of the wall is maintained at a constant temperature of $T_1 = 80$°C while the right side loses heat by convection to the surrounding air at $T_\infty = 15$°C with a heat transfer coefficient of $h = 24$ W/m^2 · °C. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the rate of heat transfer through the wall. *Answer: (c) 6030 W*

Fundamentals of Thermal-Fluid Sciences, 3rd Edition
Yunus A. Cengel, Robert H. Turner, John M. Cimbala
McGraw-Hill, 2008

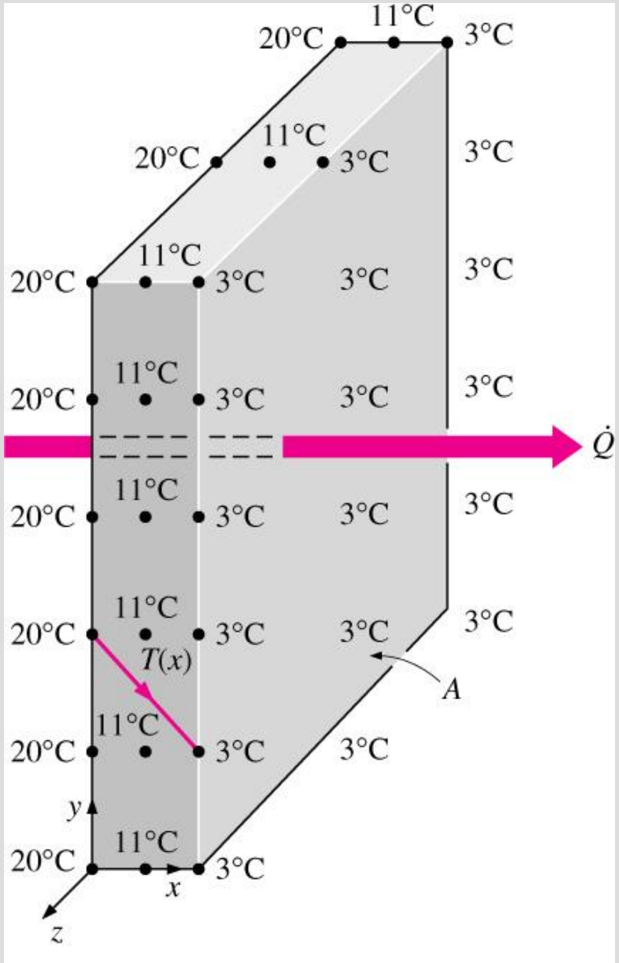
STEADY HEAT CONDUCTION

Mehmet Kanoglu

Objectives

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries
- Develop an intuitive understanding of thermal contact resistance, and circumstances under which it may be significant
- Identify applications in which insulation may actually increase heat transfer
- Analyze finned surfaces, and assess how efficiently and effectively fins enhance heat transfer
- Solve multidimensional practical heat conduction problems using conduction shape factors.

STEADY HEAT CONDUCTION IN PLANE WALLS



Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.*

The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as *T*(*x*).

$$\left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{array}\right) - \left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{array}\right)$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

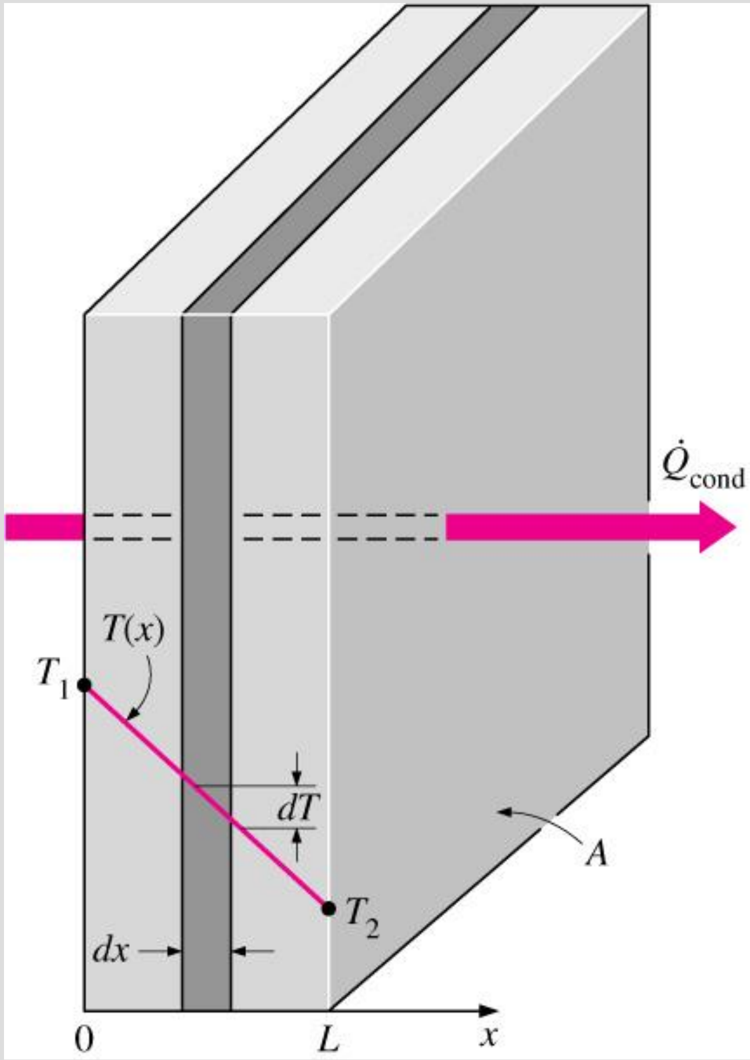
$$dE_{wall}/dt = 0$$

for *steady* operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{cond,\, wall} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Fourier's law of heat conduction



$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

Under steady conditions, the temperature distribution in a plane wall is a straight line: ***dT/dx* = const.**

Thermal Resistance Concept

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

$$R_{\text{wall}} = \frac{L}{kA} \qquad (^\circ\text{C/W})$$

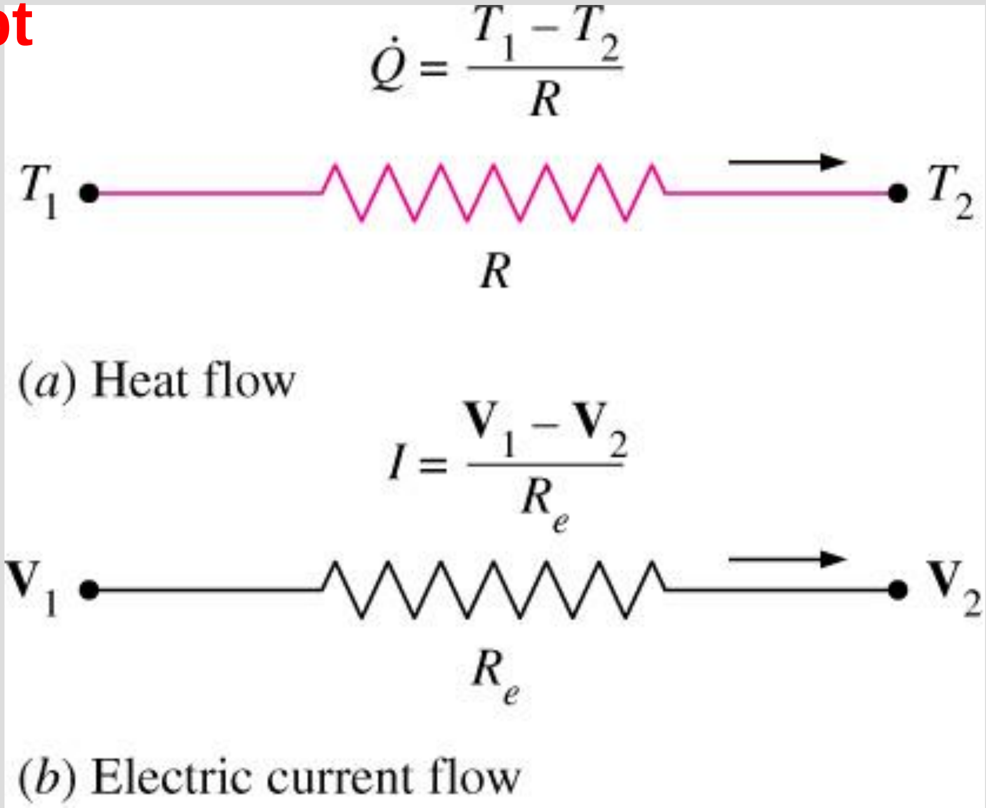
Conduction resistance of the wall: *Thermal resistance* of the wall against heat conduction.

Thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

$$R_e = L/\sigma_e A$$

Electrical resistance



Analogy between thermal and electrical resistance concepts.

rate of heat transfer → electric current
thermal resistance → electrical resistance
temperature difference → voltage difference

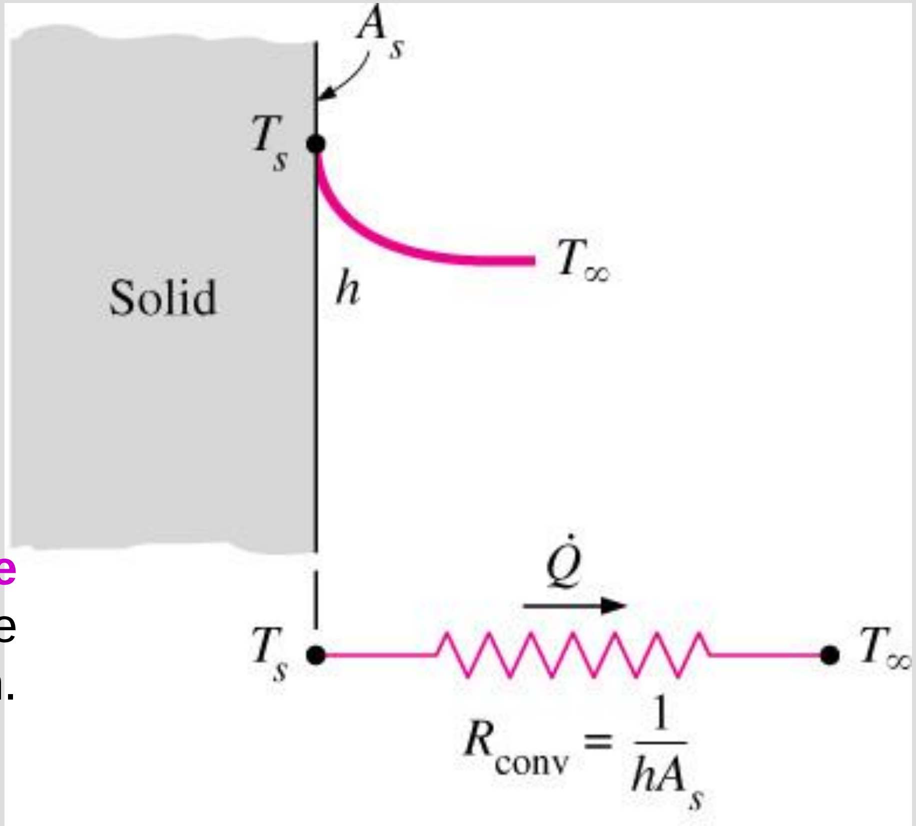
Newton's law of cooling

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad \text{(W)}$$

$$R_{conv} = \frac{1}{hA_s} \qquad \text{(°C/W)}$$

Convection resistance of the surface: *Thermal resistance* of the surface against heat convection.



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large ($h \rightarrow \infty$), the convection resistance becomes *zero* and $T_s \approx T$.

That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

This situation is approached in practice at surfaces where boiling and condensation occur.

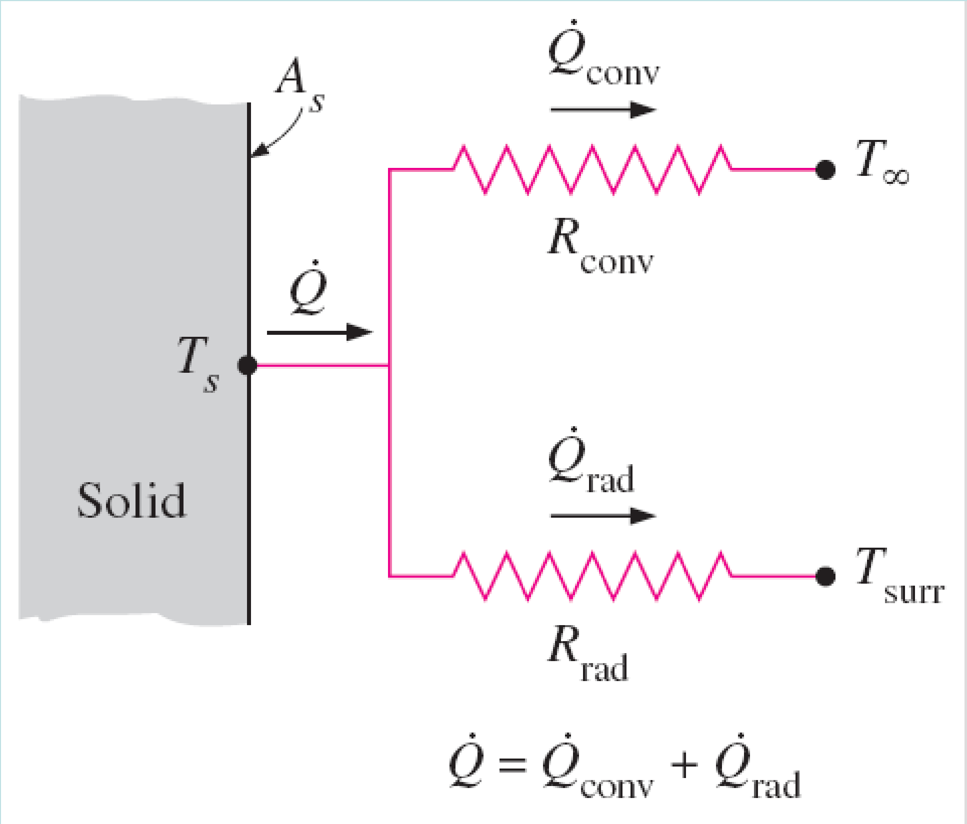
Radiation resistance of the surface: *Thermal resistance* of the surface against radiation.

Radiation heat transfer coefficient

When $T_{surr} \approx T_\infty$

$$h_{combined} = h_{conv} + h_{rad}$$

Combined heat transfer coefficient



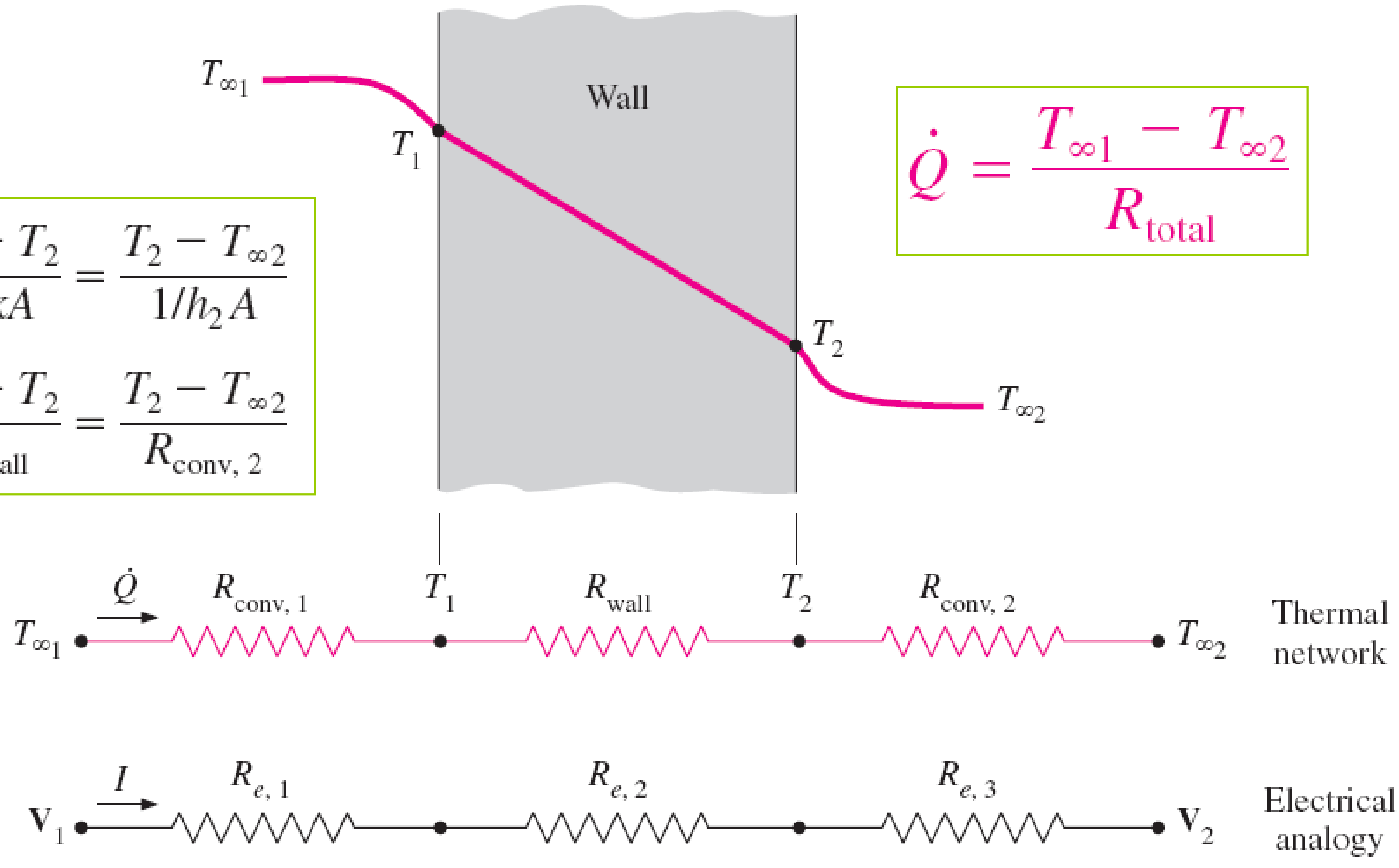
Schematic for convection and radiation resistances at a surface.

Thermal Resistance Network

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\text{C/W})$$

Temperature drop

$$\Delta T = \dot{Q}R \qquad (°C)$$

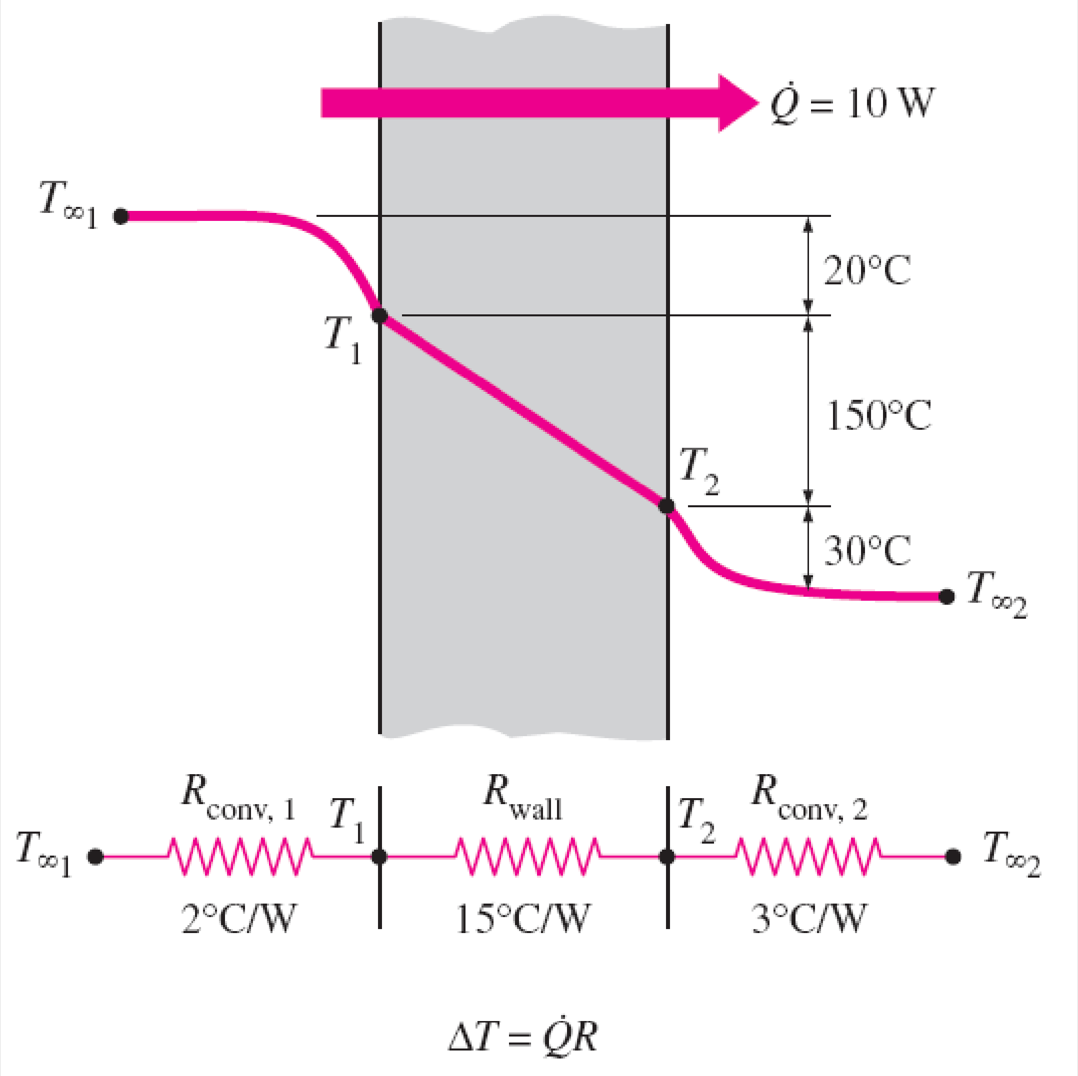
$$\dot{Q} = UA\,\Delta T \qquad (W)$$

$$UA = \frac{1}{R_{total}} \qquad (°C/K)$$

U overall heat transfer coefficient

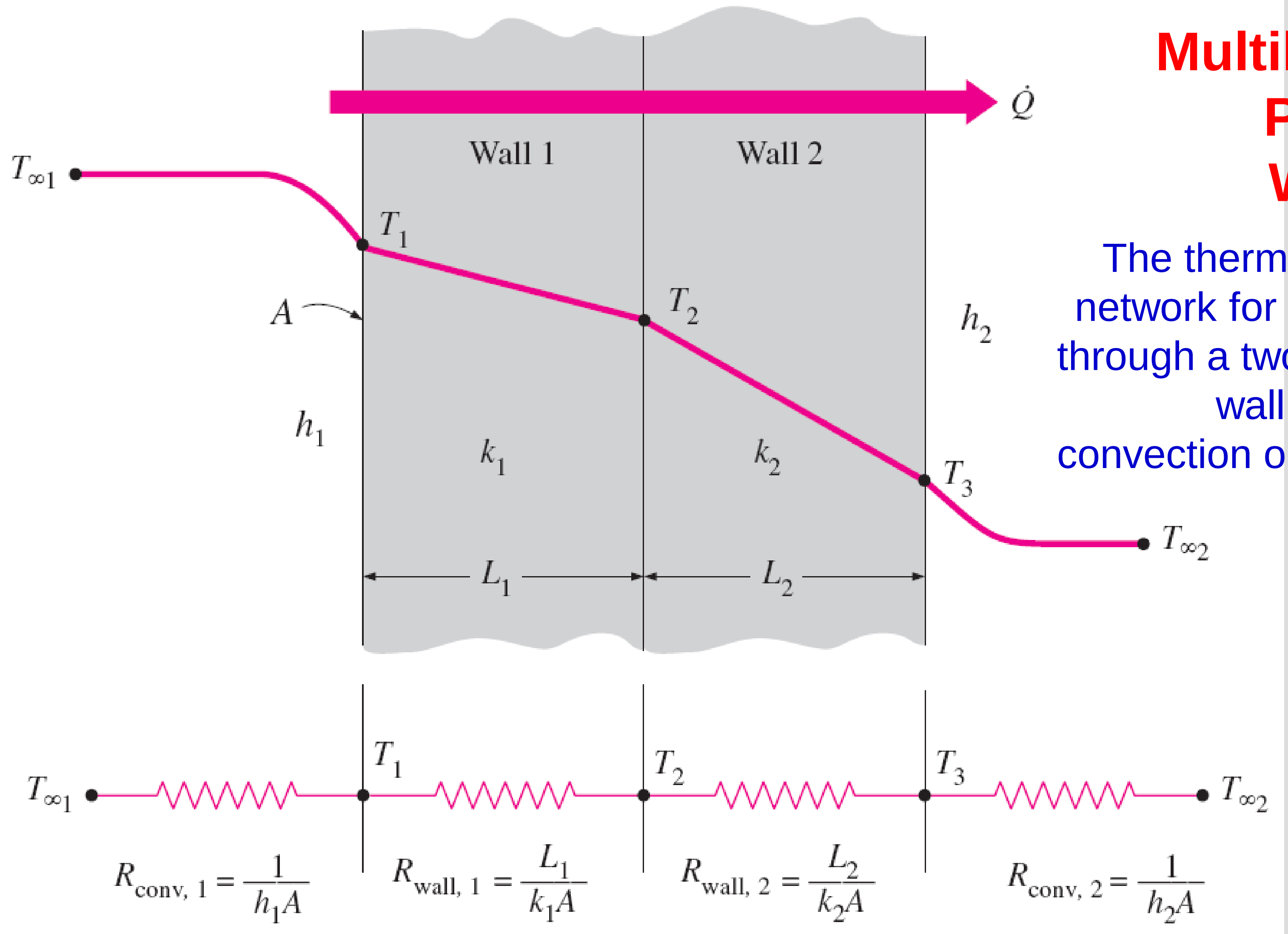
Once Q is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,\,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$



The temperature drop across a layer is proportional to its thermal resistance.

Multilayer Plane Walls



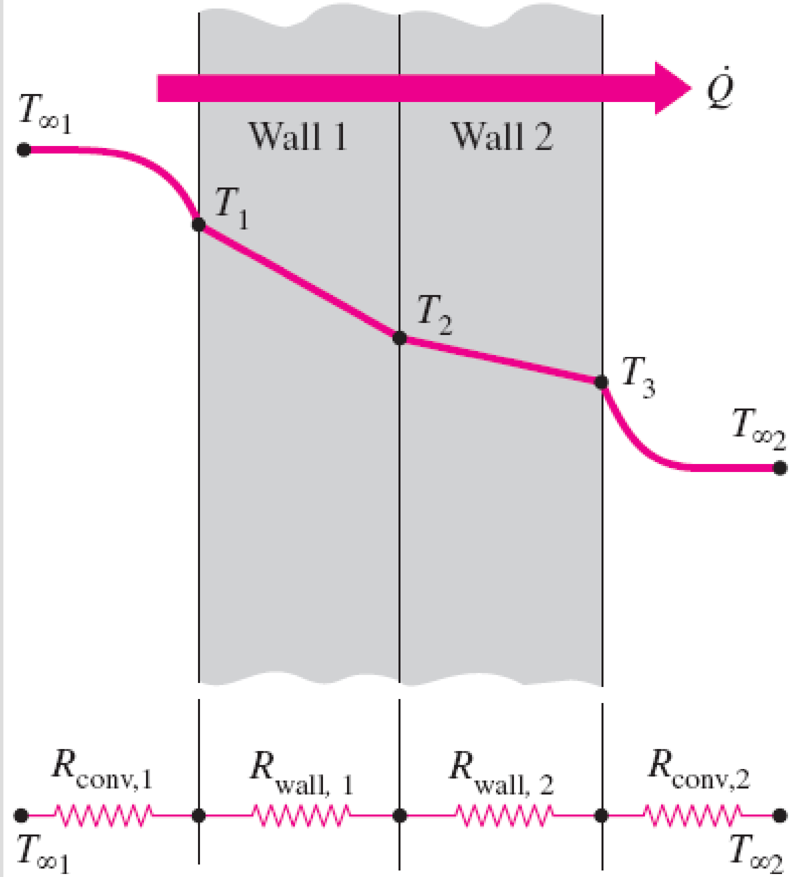
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$R_{total} = R_{conv,\,1} + R_{wall,\,1} + R_{wall,\,2} + R_{conv,\,2} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},\,1} + R_{\text{wall},\,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},\,1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

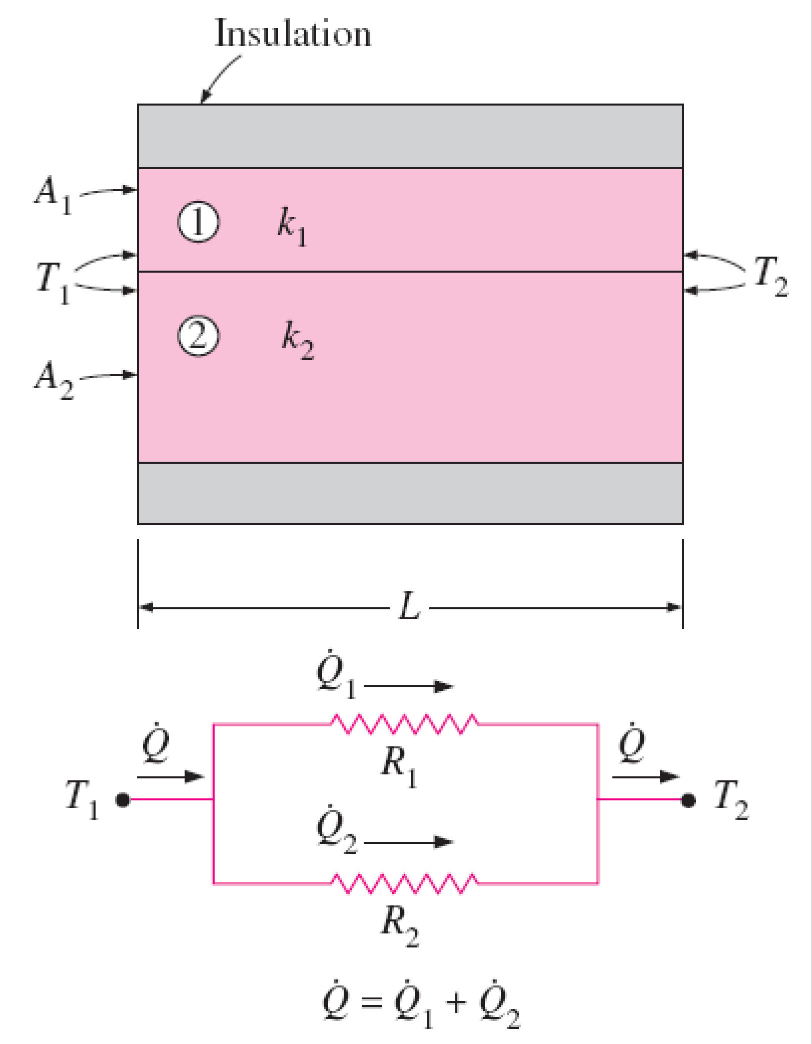
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

GENERALIZED THERMAL RESISTANCE NETWORKS

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



Thermal resistance network for two parallel layers.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

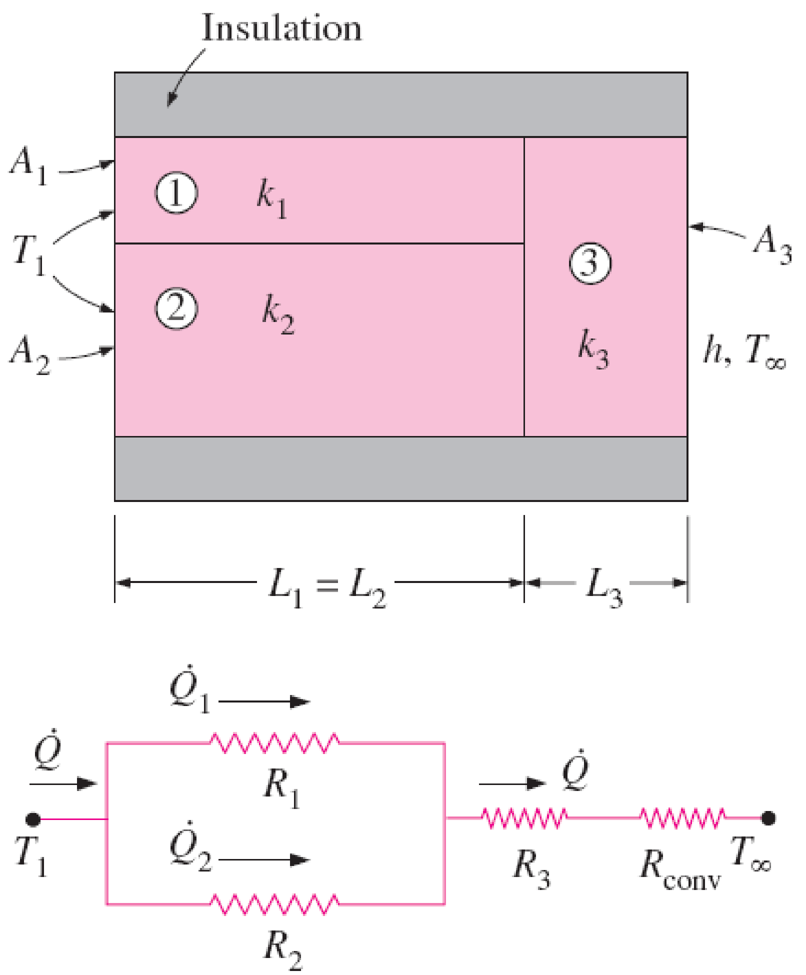
$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{conv} = \frac{1}{hA_3}$$

Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1) any plane wall normal to the x-axis is *isothermal* (i.e., to assume the temperature to vary in the x-direction only)

(2) any plane parallel to the x-axis is *adiabatic* (i.e., to assume heat transfer to occur in the x-direction only)

Do they give the same result?

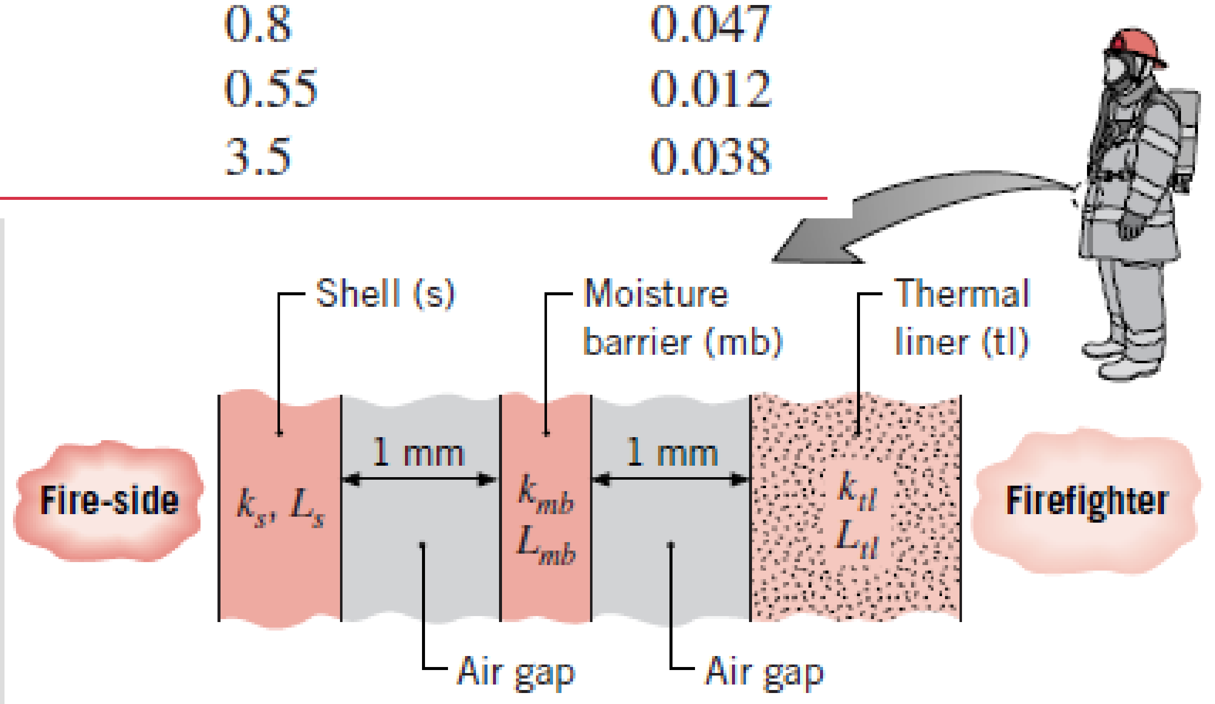


Thermal resistance network for combined series-parallel arrangement.

Example, thermal resistances

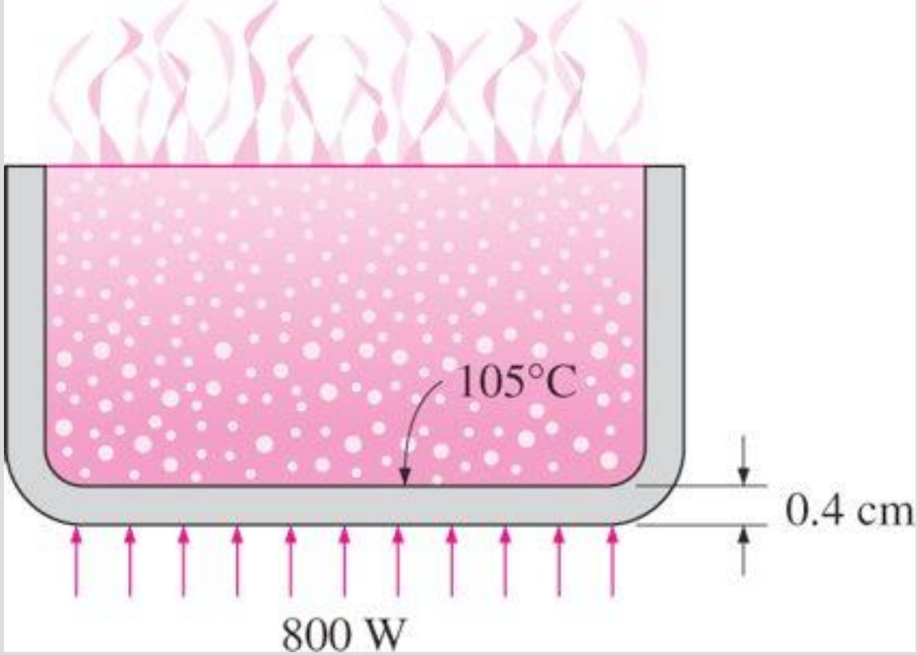
A firefighter's protective clothing, referred to as a *turnout coat,* is typically constructed as an ensemble of three layers separated by air gaps, as shown in Figure. Representative dimensions and thermal conductivities for the layers are as follows.

Layer	Thickness (mm)	k(W/m · K)
Shell (s)	0.8	0.047
Moisture barrier (mb)	0.55	0.012
Thermal liner (tl)	3.5	0.038

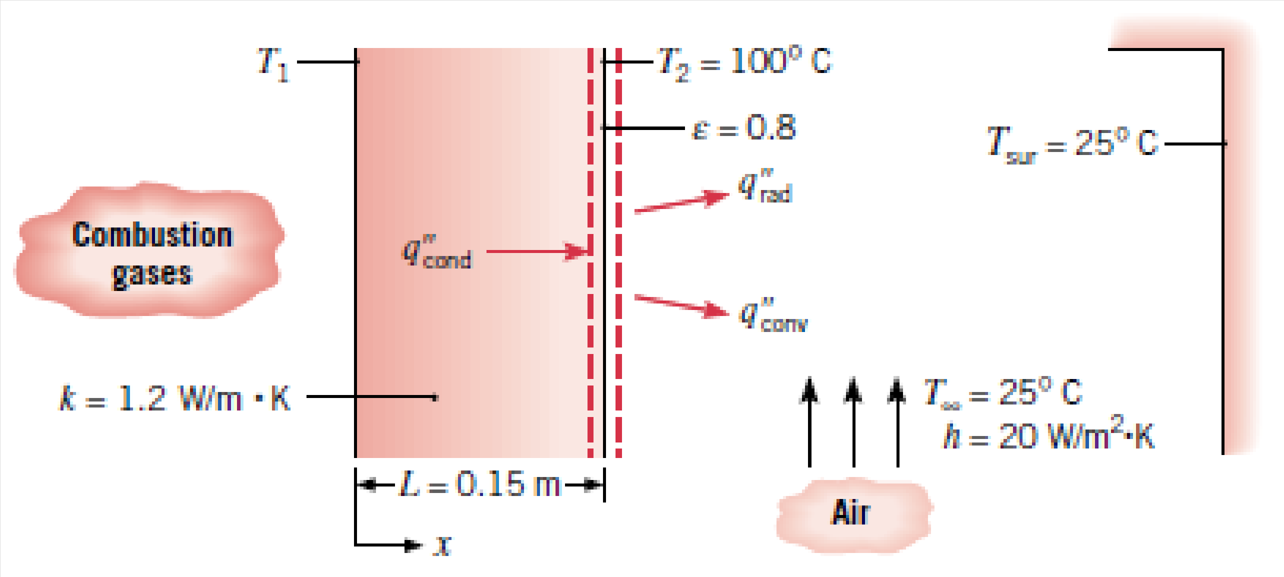


Quiz:

An aluminum pan whose thermal conductivity is 237 W/m · °C has a flat bottom with diameter 20 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 800 W. If the inner surface of the bottom of the pan is at 105°C, determine the temperature of the outer surface of the bottom of the pan.

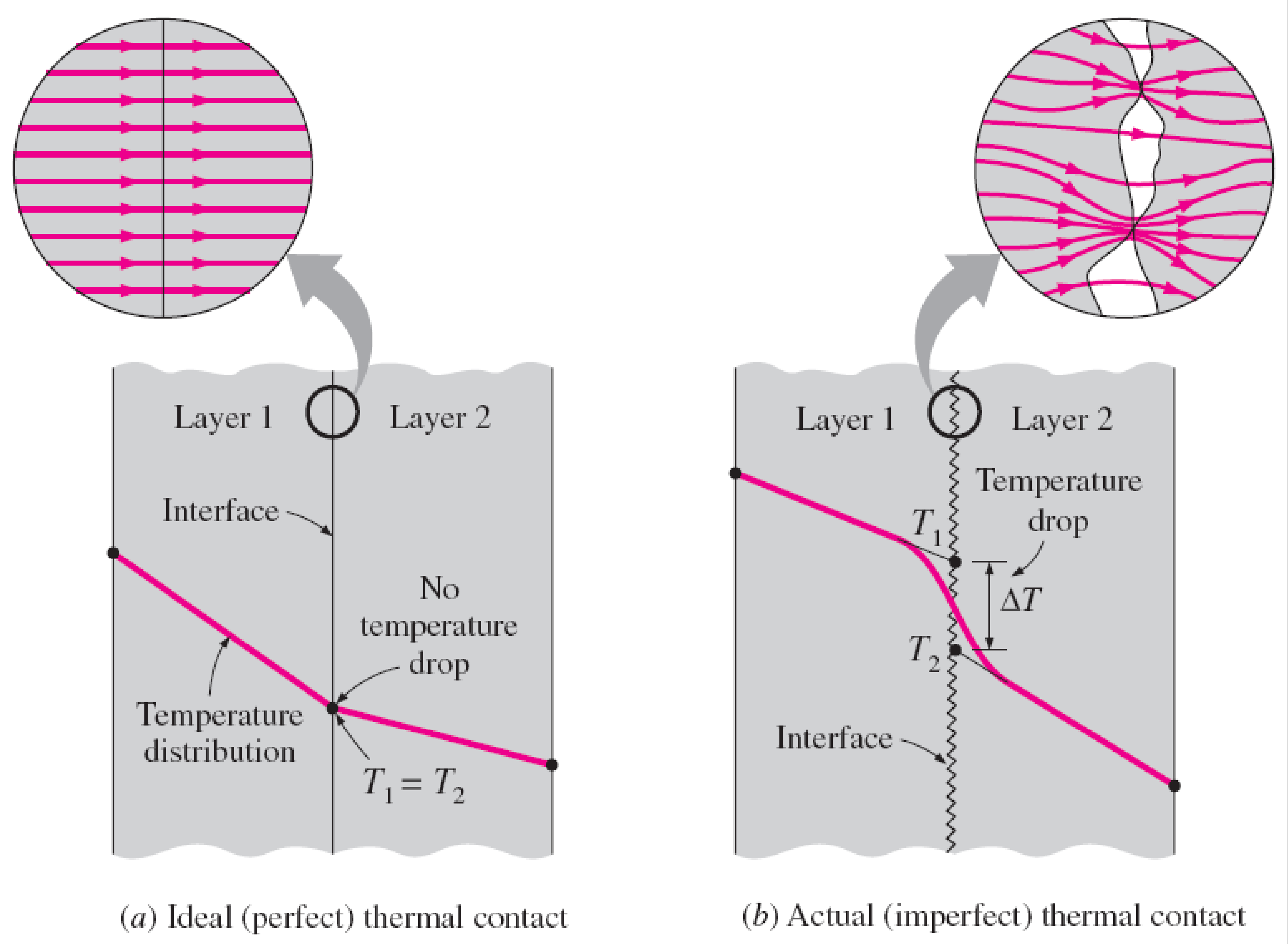


Example, energy balance



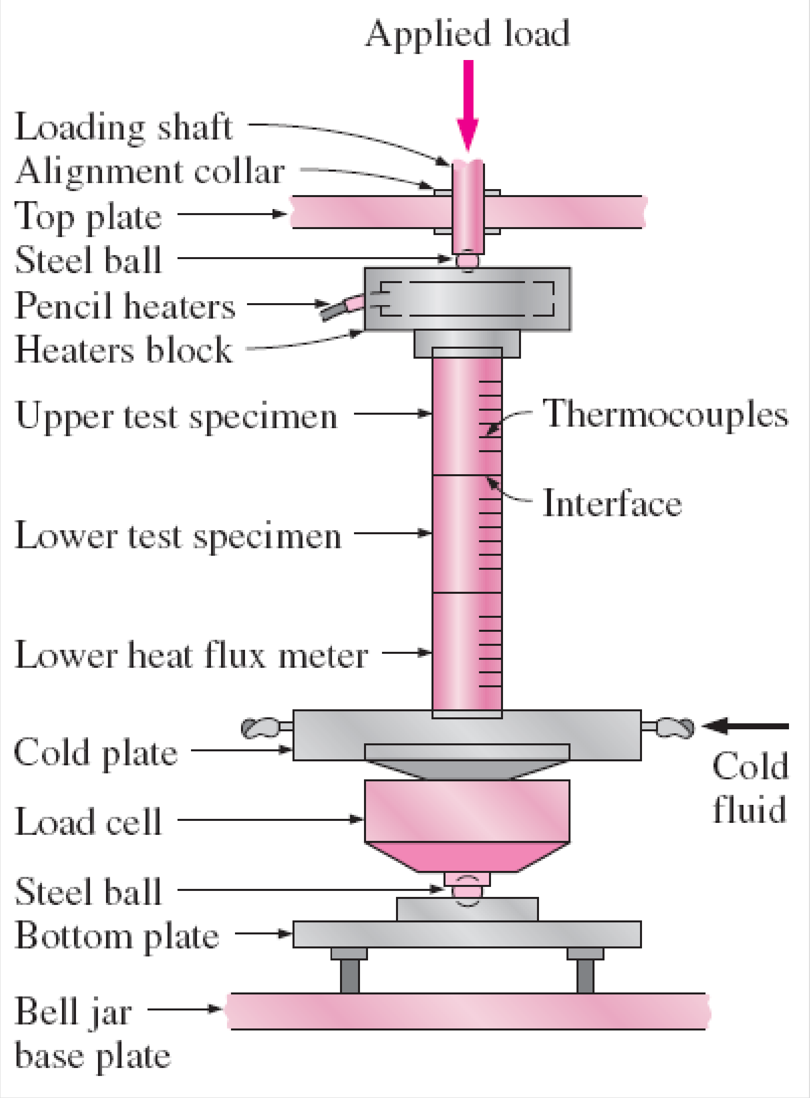
The hot combustion gases of a furnace are separated from the ambient air and its surroundings, which are at 25^{o}C, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1,2 W/m K and a surface emissivity of 0.8. Under steady-state conditions an outer surface temperature of 100^{o}C is measured. Free convection heat transfer to the air adjoining the surface is characterized by a convection coefficient of 20 W/m^2 K What is the brick inner surface temperature?

THERMAL CONTACT RESISTANCE



Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air.
- These numerous *air gaps* of varying sizes act as *insulation* because of the low thermal conductivity of air.
- Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance, R_c**.



A typical experimental setup for the determination of thermal contact resistance

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

h_c thermal contact conductance

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {^\circ}\text{C})$$

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {^\circ}\text{C/W})$$

$$R_{c,\,\text{insulation}} = \frac{L}{k} = \frac{0.01\ \text{m}}{0.04\ \text{W/m} \cdot {^\circ}\text{C}} = 0.25\ \text{m}^2 \cdot {^\circ}\text{C/W}$$

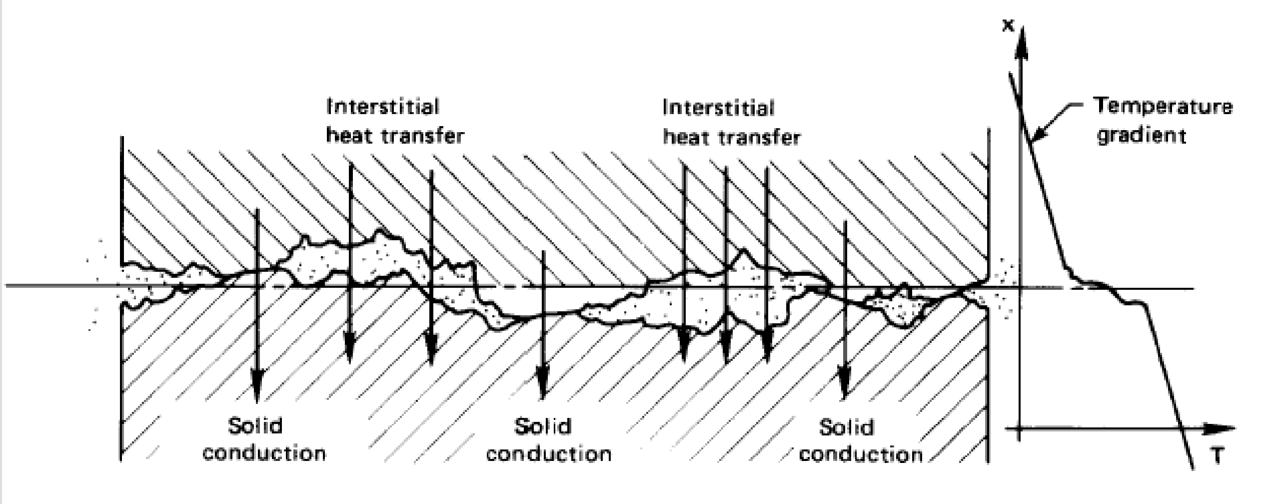
$$R_{c,\,\text{copper}} = \frac{L}{k} = \frac{0.01\ \text{m}}{386\ \text{W/m} \cdot {^\circ}\text{C}} = 0.000026\ \text{m}^2 \cdot {^\circ}\text{C/W}$$

The value of thermal contact resistance depends on:

- *surface roughness,*
- *material properties,*
- *temperature* and *pressure* at the interface
- *type of fluid* trapped at the interface.

Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be disregarded for poor heat conductors such as insulations.

THERMAL CONTACT RESISTANCE



The interfacial conductance, h_c, depends on the following factors:

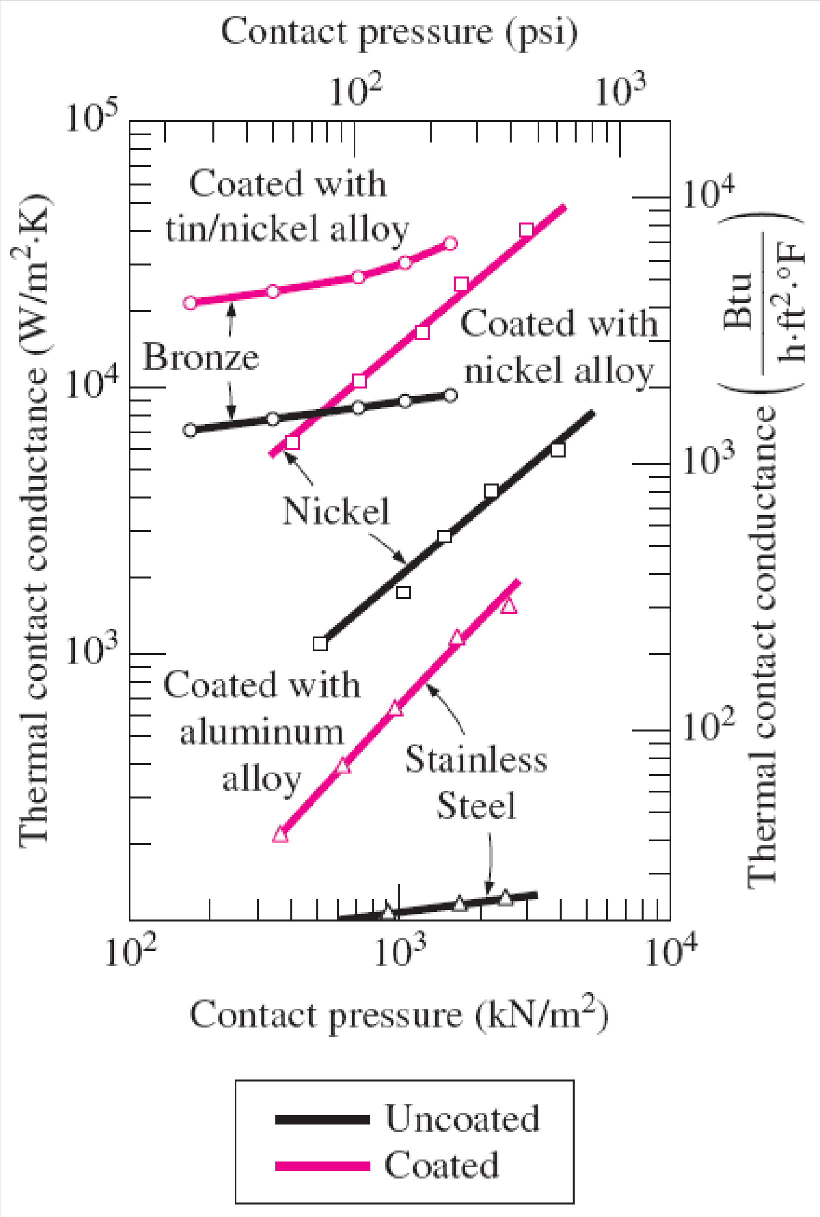
- The surface finish and cleanliness of the contacting solids.
- The materials that are in contact.
- The pressure with which the surfaces are forced together. This may vary over the surface, for example, in the vicinity of a bolt.
- The substance (or lack of it) in the interstitial spaces. Conductive shims or fillers can raise the interfacial conductance.
- The temperature at the contact plane.

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, 1969).

Fluid at the interface	Contact conductance, h_c, $W/m^2 \cdot K$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

The thermal contact resistance can be minimized by applying

- a *thermal grease* such as silicon oil
- a *better conducting gas* such as helium or hydrogen
- a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum



Effect of metallic coatings on thermal contact conductance

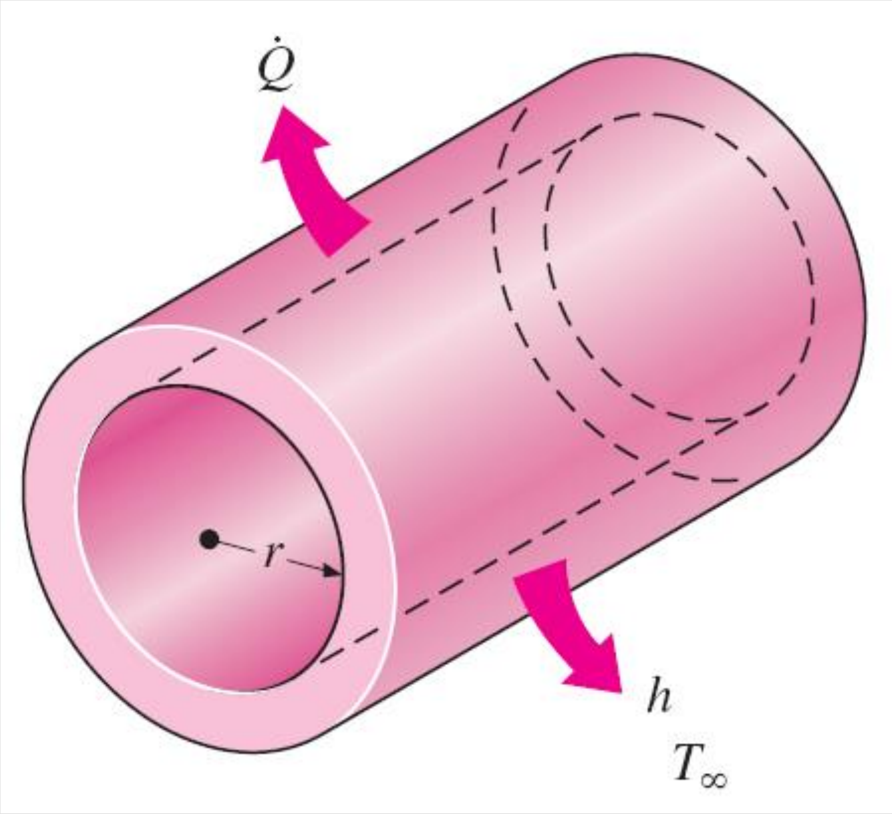
TABLE 17–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m² · °C
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel– Aluminum		20–30	20	10 20	2900 3600
Stainless steel– Aluminum		1.0–2.0	20	10 20	16,400 20,800
Steel Ct-30– Aluminum	Ground	1.4–2.0	20	10 15–35	50,000 59,000
Steel Ct-30– Aluminum	Milled	4.5–7.2	20	10 30	4800 8300
Aluminum-Copper	Ground	1.17–1.4	20	5 15	42,000 56,000
Aluminum-Copper	Milled	4.4–4.5	20	10 20–35	12,000 22,000

The *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure.*

HEAT CONDUCTION IN CYLINDERS AND SPHERES



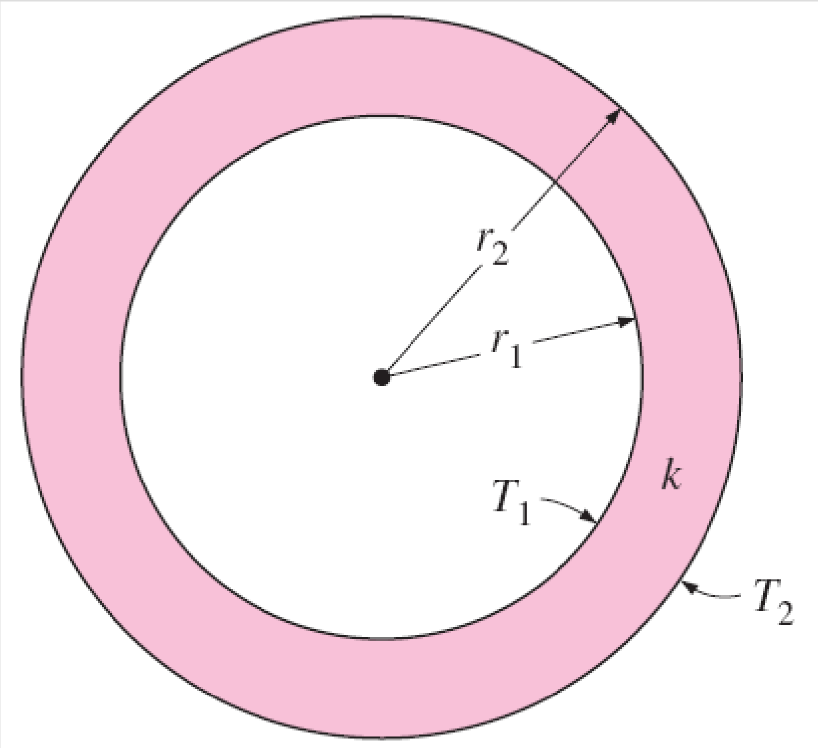
Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

Heat transfer through the pipe can be modeled as *steady* and *one-dimensional*.

The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$.

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.



A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

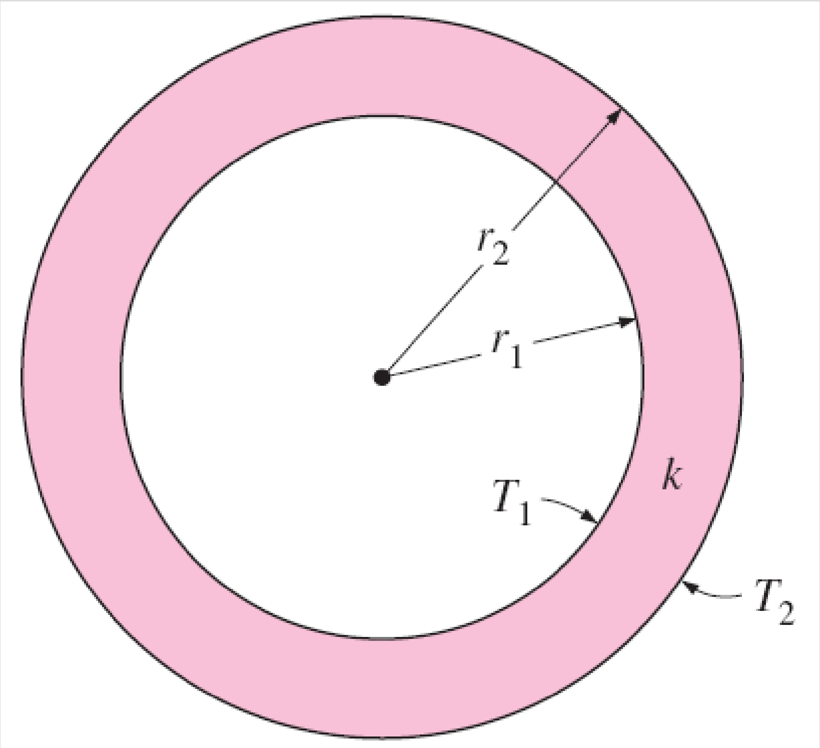
$$T(r = r_1) = T_1 \quad and \quad T(r = r_2) = T_2$$

$$T = T_1 + \frac{T_1 - T_2}{\ln(r_1 / r_2)} \ln \frac{r}{r_1}$$

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\, dr = -\int_{T=T_1}^{T_2} k\, dT$$

$$A = 2\pi r L$$



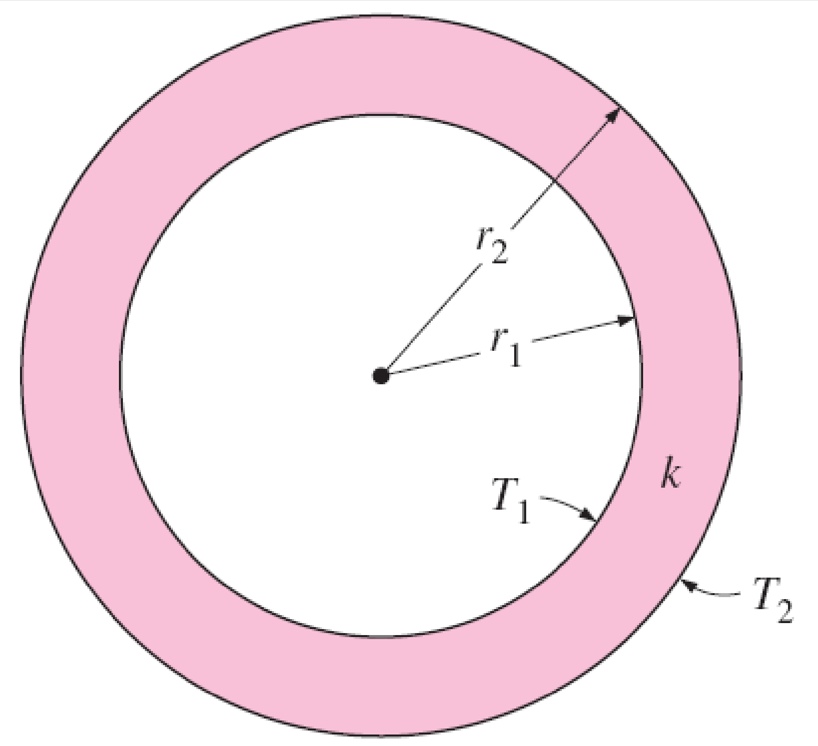
$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk \frac{T_1 - T_2}{\ln(r_2/r_1)} \quad \text{(W)}$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \quad \text{(W)}$$

A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

Conduction resistance of the cylinder layer



$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

$$T(r=r_1)=T_1 \quad and \quad T(r=r_2)=T_2$$

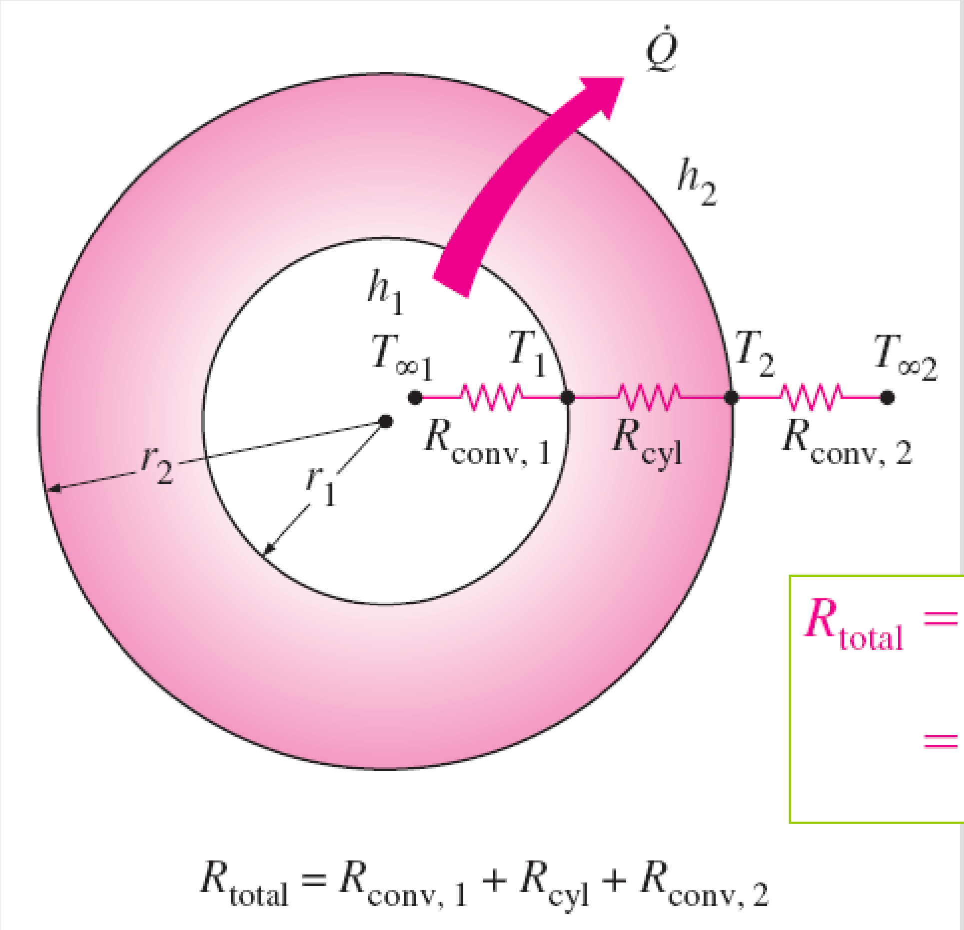
$$T=T_1+\frac{T_1-T_2}{1/r_2-1/r_1}\left(\frac{1}{r_1}-\frac{1}{r}\right)$$

A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, sph}}=\frac{T_1-T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}}=\frac{r_2-r_1}{4\pi r_1 r_2 k}=\frac{\text{Outer radius}-\text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

Conduction resistance of the spherical layer



The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

for a *cylindrical* layer

$$R_{total} = R_{conv,\,1} + R_{cyl} + R_{conv,\,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *spherical* layer

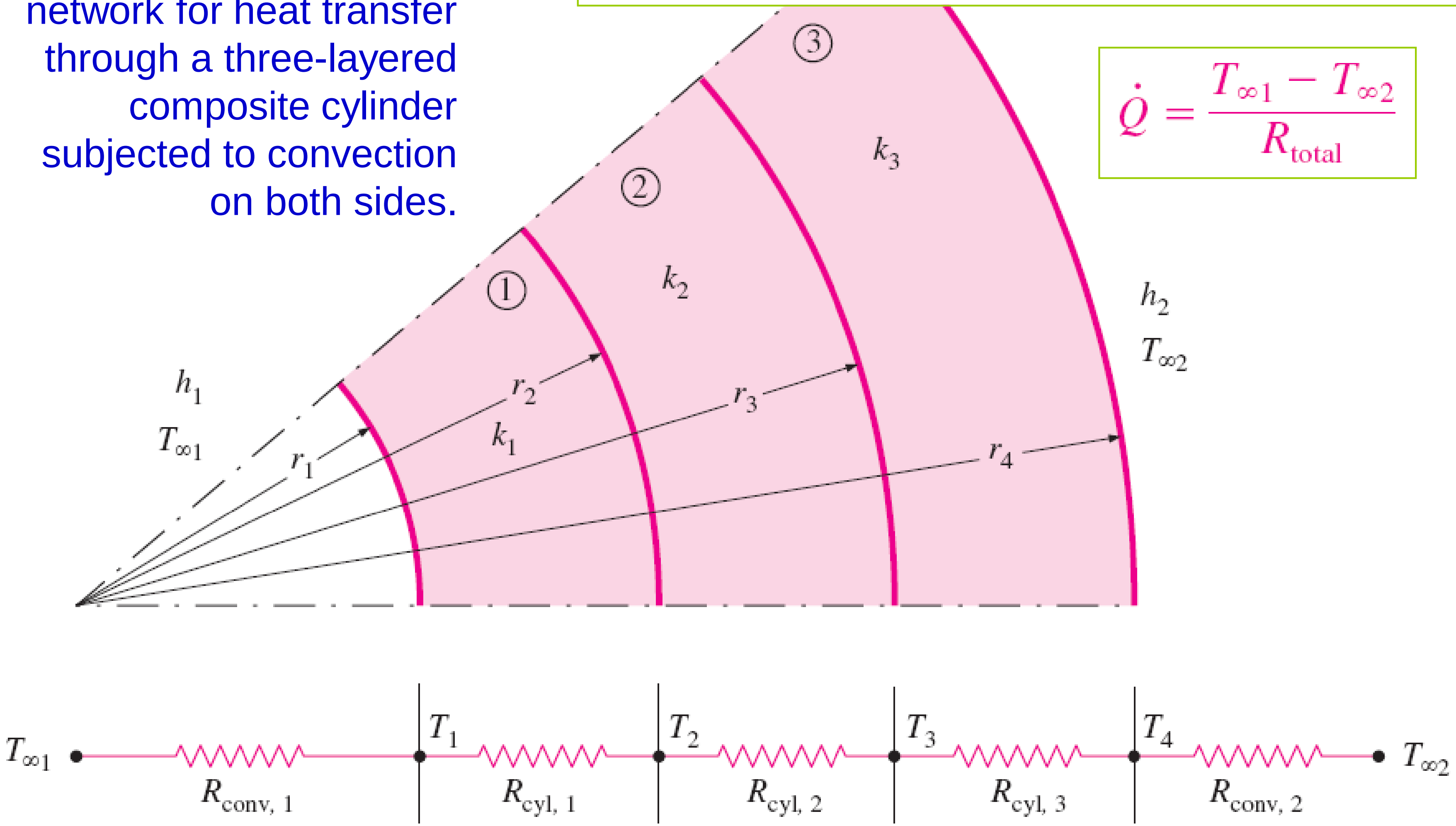
$$R_{total} = R_{conv,\,1} + R_{sph} + R_{conv,\,2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

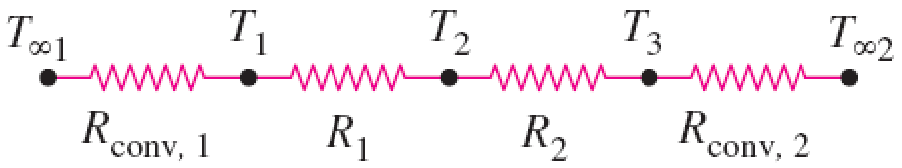
Multilayered Cylinders and Spheres

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$





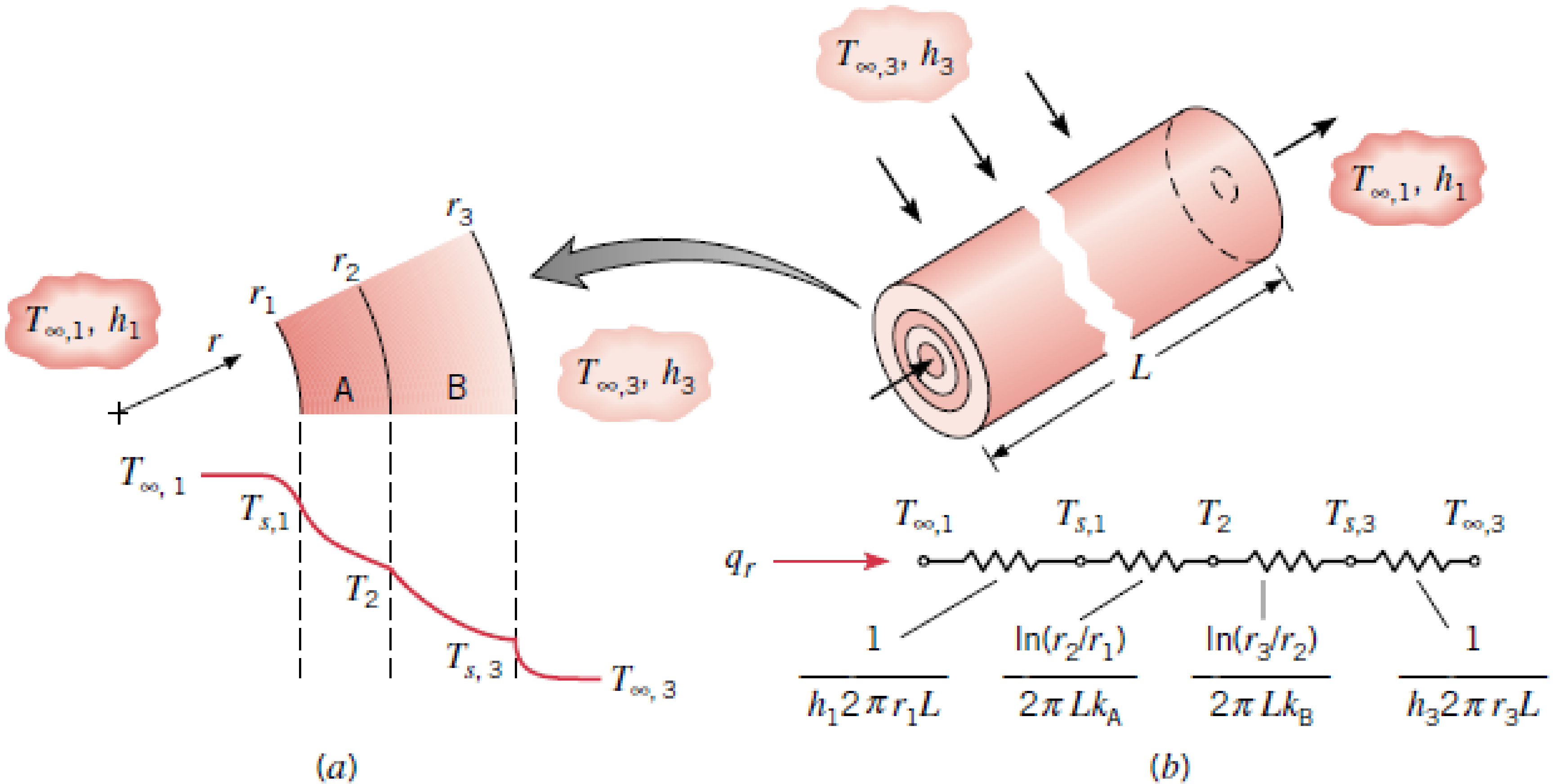
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1} = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}} = \cdots$$

The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

Once heat transfer rate Q has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$

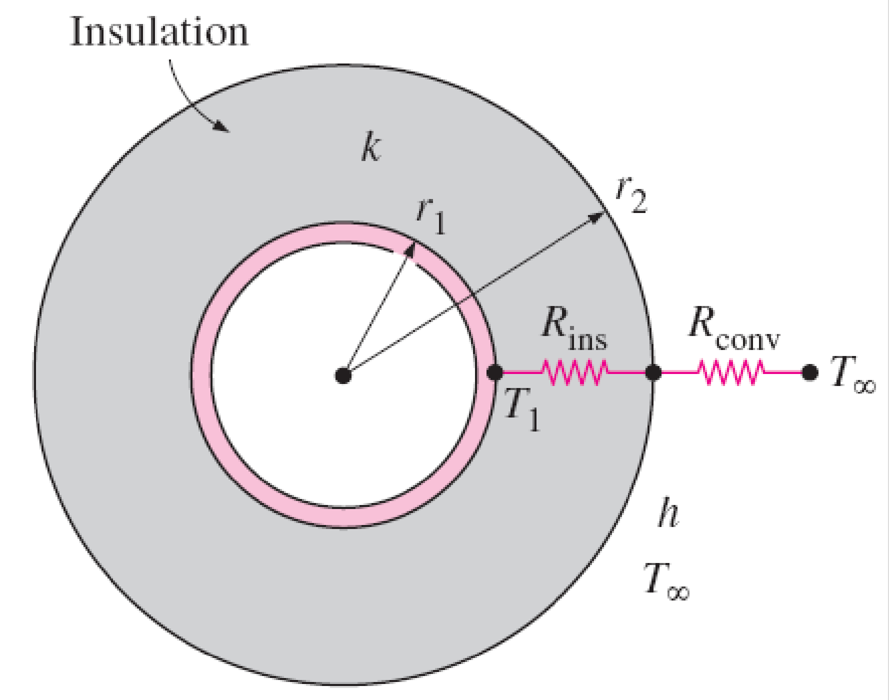

T∞,3, h3
T∞,1, h1
r3
r2
r1
T∞,1, h1
r
A
B
T∞,3, h3
L
T∞,1
Ts,1
T2
Ts,3
T∞,3
T∞,1
Ts,1
T2
Ts,3
T∞,3
qr
1 / h1 2π r1 L
ln(r2/r1) / 2π L kA
ln(r3/r2) / 2π L kB
1 / h3 2π r3 L
(a)
(b)

CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

The heat transfer from the pipe may increase or decrease, depending on which effect dominates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi L k} + \dfrac{1}{h(2\pi r_2 L)}}$$

The critical radius of insulation for a cylindrical body:

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)}$$

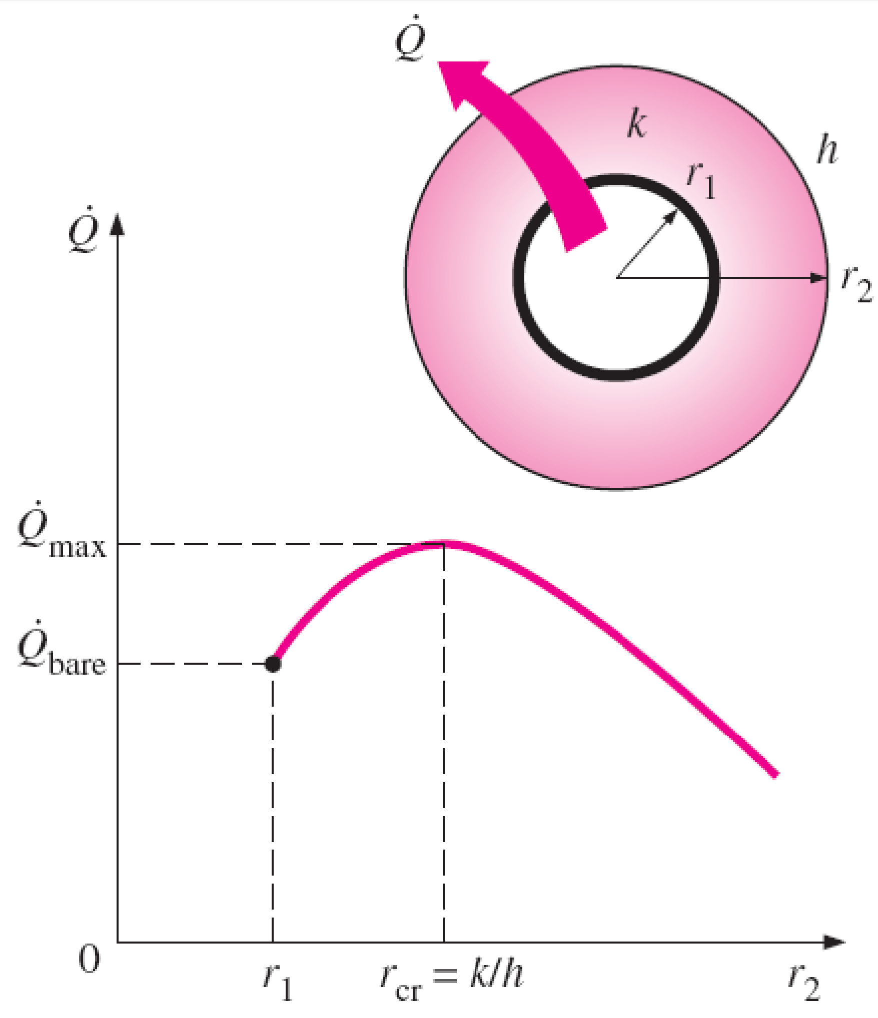
The critical radius of insulation for a spherical shell:

$$r_{\text{cr, sphere}} = \frac{2k}{h}$$

The largest value of the critical radius we are likely to encounter is

$$r_{\text{cr, max}} = \frac{k_{\text{max, insulation}}}{h_{\text{min}}} \approx \frac{0.05 \text{ W/m} \cdot °\text{C}}{5 \text{ W/m}^2 \cdot °\text{C}} = 0.01 \text{ m} = 1 \text{ cm}$$

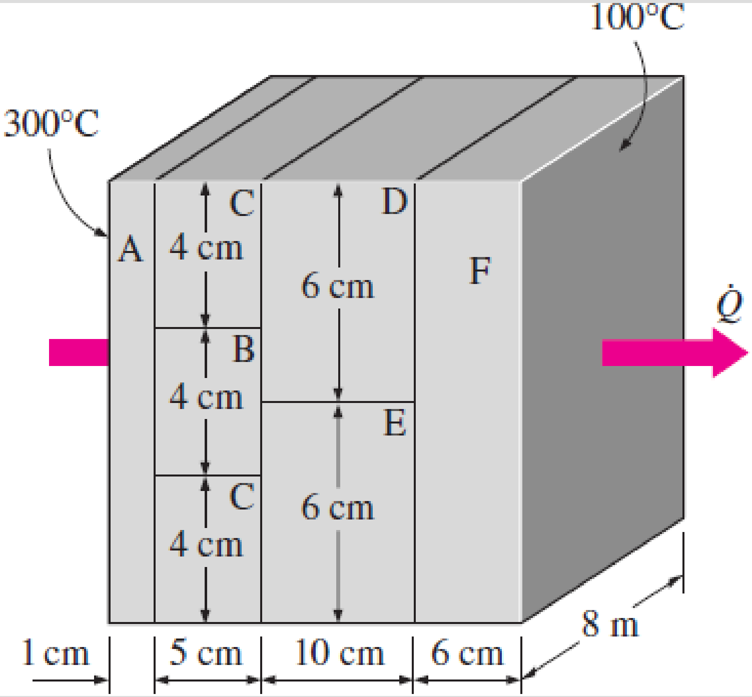
We can insulate hot-water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.



The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{cr}$.

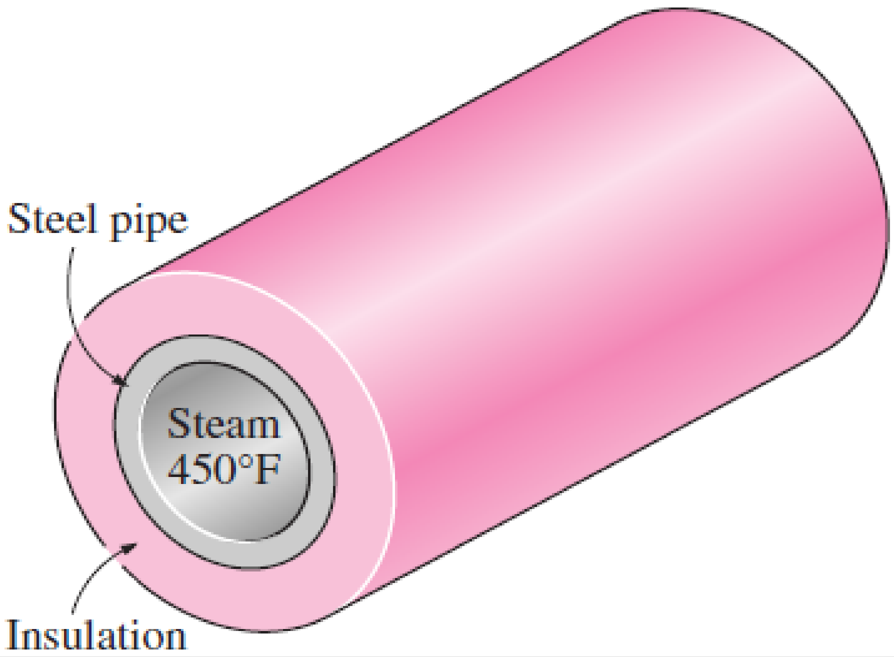
Examples, Cengel

3–57 Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m · °C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet; and (*c*) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.



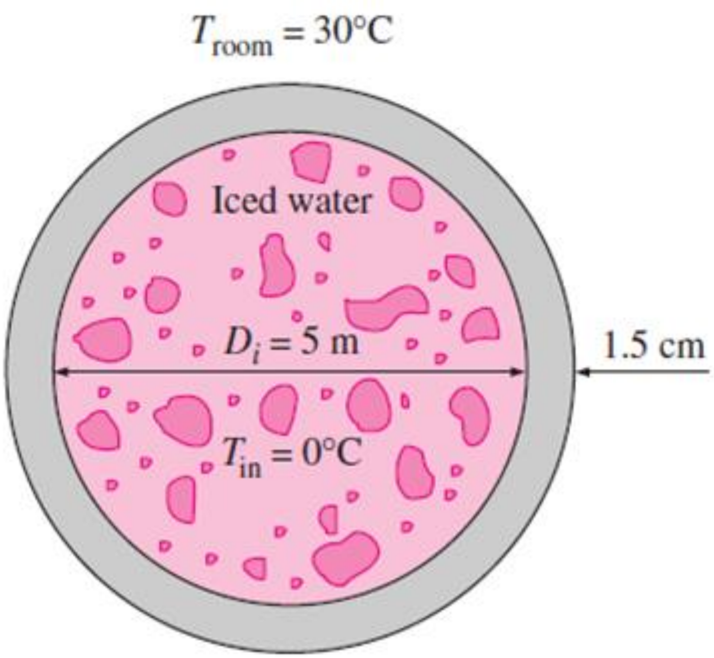
Example, 3-75E, Cengel

3–75E Steam at 450°F is flowing through a steel pipe (k = 8.7 Btu/h · ft · °F) whose inner and outer diameters are 3.5 in. and 4.0 in., respectively, in an environment at 55°F. The pipe is insulated with 2-in.-thick fiberglass insulation (k = 0.020 Btu/h · ft · °F). If the heat transfer coefficients on the inside and the outside of the pipe are 30 and 5 Btu/h · ft^2 · °F, respectively, determine the rate of heat loss from the steam per foot length of the pipe. What is the error involved in neglecting the thermal resistance of the steel pipe in calculations?



Problem

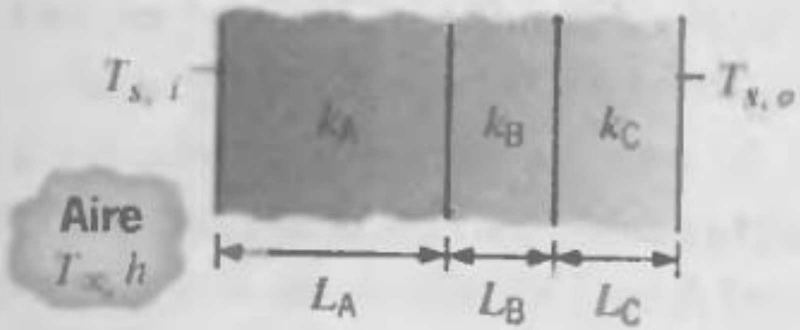
A 5-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at 0°C. The tank is located in a room whose temperature is 30°C. The walls of the room are also at 30°C. The outer surface of the tank is black (emissivity $\varepsilon = 1$), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m^2 · °C and 10 W/m^2 · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg.



$$h_{rad} = \varepsilon\sigma(T^2 + T_{surr}^2)(T + T_{surr})$$

Quiz

La pared compuesta de un horno consiste en tres materiales, dos de los cuales son de conductividad térmica conocida, $k_A = 20\ W/m \cdot K$ y $k_C = 50\ W/m \cdot K$, y de espesor conocido, $L_A = 0.30\ m$ y $L_C = 0.15\ m$. El tercer material, B, que se intercala entre los materiales A y C, es de espesor conocido, $L_B = 0.15\ m$, pero de conductividad térmica, k_B, desconocida.



En condiciones de operación de estado estable, las mediciones revelan una temperatura de la superficie externa $T_{s,o} = 20°C$, una temperatura de la superficie interna $T_{s,i} = 600°C$, y una temperatura del aire del horno $T_\infty = 800°C$. Se sabe que el coeficiente de convección interior h es $25\ W/m^2 \cdot K$. ¿Cuál es el valor de k_B?