NEURAL INVERSE OPTIMAL BLOCK CONTROL: APPLICATION TO A 2-DOF HELICOPTER

CONTROL POR BLOQUES ÓPTIMO INVERSO NEURONAL: APLICACIÓN A UN HELICÓPTERO DE 2-DOF

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Abstract: This paper provides a solution to the trajectory-tracking problem in discrete-time of non-linear systems subject to parametric variations, using the block-based approach with a state observer. The problem of controlling a two-degree of freedom (2-DOF) helicopter is solved, where the identification process is delegated to a recurrent high-order neural network in discrete-time, trained online by means of an Extended Kalman filter. Additionally, an optimal inverse control in discrete-time based in passivity is selected, to obtain a smooth control law capable of keeping the energy dissipation in the system to a minimum through the satisfaction of a cost functional. The results are presented in simulation and real-time implementation.

Keywords: 2-DOF helicopter, block control, discrete-time inverse optimal control, recurrent high order neural networks, real-time implementation.

Resumen: El presente trabajo proporciona una solución al seguimiento de trayectoria en tiempo discreto de sistemas no-lineales sometidos a variaciones paramétricas, mediante el enfoque del control por bloques basado en observador de estados. Se resuelve el problema de control de un helicóptero 2-DOF, donde el proceso de identificación se delega a una red neuronal recurrente de alto orden en tiempo discreto, entrenada en línea por medio del Filtro de Kalman Extendido. Adicionalmente, se selecciona un control óptimo inverso en tiempo discreto por pasividad, para obtener una ley de control suave capaz de mantener al mínimo el gasto energético en el sistema a través de la satisfacción de un funcional de costo. Los resultados se presentan en simulación e implementación en tiempo real.

Palabras clave: Helicóptero de 2-*DOF*, control por bloques, control óptimo inverso en tiempo discreto, redes neuronales recurrentes de alto orden, tiempo real.

1. INTRODUCTION

The development of new strategies and algorithms around control theory, has become a trend for the treatment of a variety of dynamic systems ranging from biology to applied engineering. In the latter, interest has been focused on the modelling and control of electromechanical systems of wide mathematical complexity such as drones, planes, rockets, and helicopters; either in theoretical developments or practical applications, this particular type of systems constitutes a remarkable challenge. Among these systems, the electromechanical helicopter model stands out due to its highly-nonlinear dynamics, which can be used to test a wide variety of techniques based on state implementation such as inverse optimal control, among others. Work has been carried out focused on the control of this particular system type, as can be seen in (Kumar *et al.,* 2016) where a linear approximation of a 2-DOF model is formulated by designing a LQR control. A set of different nonlinear tools for controlling a 2-DOF nonlinear model, such as neural backstepping are presented in (Hernandez-Gonzalez, *et al.,* 2012). In a more recent work, the combination of higher order sliding mode control strategy, with the optimal control technique, aiming to solve a trajectory-tracking problem was presented in (Boukadida *et al.,* 2019). By making use of the Sylvester equation and metaheuristic algorithms, the authors present an optimal solution to the tracking problem of the 2-DOF helicopter system, namely, the Quanser Aero helicopter. In (Steinbusch and Reyhanoglu, 2019) the solution to the same problem relies on the use of Sliding Mode Control and dynamic filters that produce the desired sliding surface. Likewise, the sliding mode control technique alongside the generalized observer proportional integral observer was addressed in (Rojas-Cubides *et al.,* 2019), here the trajectory tracking of the 2-DOF helicopter subject to plant parameter variations and faults present in the actuators, with the combination of both techniques is solved. On the other hand, as a method to simplify the handling of systems with a relative degree major to one, and according to (Drakunov, 1990), a block control scheme is proposed. This scheme allows the original problem to be divided into smaller independent sub-problems, and presents the definition of a fictitious control prescript in the state of the preceding block. Additionally, the existence of parametric variations has been discussed in the last years by researchers of the same area, and has given a large number of mathematical tools, among which stand out adaptable control, state observers, and artificial neural networks. The latter, inspired in biology and initially used for pattern identification, have been constantly studied because of its capacity to imitate nonlinear systems dynamics (Kosmatopoulos, 1995). At the same time, neural networks have been used as state observers, incorporating among other methods, the *Extended Kalman Filter* (EKF) as a recursive algorithm for the estimation of its synaptic weights (Puskorius and Feldkamp, 1994; Simon, 2002; Haykin, 2004).

In this work, a *Recurrent High-Order Neural Network* (RHONN) is proposed as a discrete-time neural identifier for the Quanser 2-DOF helicopter model, with an *Extended Kalman Filter* to adapt its weights, ensuring the system constant evolution and eliminating excessive storage of data required for state estimation. Based on this approach, the system is represented in the block controllable form, applying on its last block, a discrete-time inverse optimal controller. This approach depends on the knowledge of a discrete-time control Lyapunov function, to minimize a cost function, therefore avoiding to solve the associated Hamilton–Jacobi–Bellman (HJB) equation.

In what follows, the structure of this paper is described briefly: In Section 2 a few mathematical aspects associated with discrete-time high-order neural networks and EKF training algorithm are given. From *Lagrange* formulation, in Section 3 a mathematical model for the system is derived, and by means of a numerical discretization scheme, a RHONN model is proposed. Using the discrete-time neural model of the system, in Section 4, a detailed derivation of the block controllable form is shown, considering an inverse optimal controller to achieve stabilization and trajectory tracking via feedback passiveness. Then, in Section 5, the results of simulation and real-time implementation for trajectory tracking are shown. Finally, the conclusions are presented in the last section.

2. PRELIMINARIES

Neural block control adaptation involves in its structure a special kind of neural network, and mathematical methods for variable estimation; hence, a few mathematical concepts are covered in the following section for the correct understanding of this paper.

2.1 Discrete-time Recurrent High-Order Neural Network

The formation of high-order terms by means of double, triple, etc. interactions between the outputs of the neurons, conform a very singular and useful kind of neural networks, known as Recurrent High-Order Neural Networks (RHONN), which, given its nonlinear nature are very useful in nonlinear systems identification (Rovithakis and Christodoulou, 2012). Based in (Sanchez, *et al.*, 2008), discrete-time systems with n state variables and m inputs can be represented by RHONN with n neurons and m inputs, where each neuron corresponds an estimate system state, whose difference equation is given by

$$\hat{x}_{i,k+1} = \left[\hat{\mathbf{w}}_{i,k}\right]^T \mathbf{z}_k(\mathbf{y}_k), \quad i = 1,2,\ldots,n \tag{1}$$

with

$$\hat{\mathbf{w}}_{i,k} = \mathbf{w}^*_{i,k} + \tilde{\mathbf{w}}_{i,k} = \left[\hat{w}_{i1,k}, \ldots, \hat{w}_{iL,k}\right]^T \quad i = 1,2,\ldots,n \tag{2}$$

$$\mathbf{y}_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \\ \vdots \\ y_{n,k} \\ y_{n+1,k} \\ \vdots \\ y_{n+m,k} \end{bmatrix} = \begin{bmatrix} s(x_{1,k}) \\ s(x_{2,k}) \\ \vdots \\ s(x_{n,k}) \\ u_{1,k} \\ \vdots \\ u_{m,k} \end{bmatrix},$$

$$\mathbf{z}_k = \begin{bmatrix} z_{1,k} \\ z_{2,k} \\ \vdots \\ z_{L,k} \end{bmatrix} = \begin{bmatrix} \prod_{j \in I_1} y_{j,k}^{d_j(\rho)} \\ \prod_{j \in I_2} y_{j,k}^{d_j(\rho)} \\ \vdots \\ \prod_{j \in I_L} y_{j,k}^{d_j(\rho)} \end{bmatrix}, \tag{3}$$

where $\hat{\mathbf{w}}_{i,k} \in \mathbb{R}^L$ is the sum between optimal weights vector $\mathbf{w}_{i,k}^* \in \mathbb{R}^L$ and the estimation error vector $\tilde{\mathbf{w}}_{i,k} \in \mathbb{R}^L$; $\mathbf{y}_k \in \mathbb{R}^L$ is the network input vector, which can be conformed either by the *i-th* real system state $x_{i,k}$ or the *i-th* estimated system state by the neural network $\hat{x}_{i,k}$; $\mathbf{u}_k \in \mathbb{R}^m$ is the external input vector; $\mathbf{z}_k$ is the set of $L = n + m$ high-order terms where $d_j(\rho)$ are non-negative integers; and $s(\cdot)$ is a monotonically increasing and differentiable activation function, typically a sigmoid function whose mathematical expression is given by

$$s(x_{i,k}) = \frac{\sigma}{1 + e^{-\beta x_{i,k}}} - \gamma$$
$$i = 1, 2, \ldots, n$$

where σ indicates saturation, β the slope, and γ the shift from the origin.

2.2 Global training - Extended Kalman Filter

The solution for the training problem using the well-known Kalman filter theory, is described as the estimation of the synaptic weights through the minimization of the quadratic error between the real output and the estimated one, ensuring that the tracking error is semiglobally uniformly ultimately bounded, therefore causing the weights to remain bounded as well, (Haykin, 2004; Sanchez, 2008). The algorithm for the recursive training is reduced to the computation of the following equations:

$$\mathbf{A}_k = \left[\mathbf{R}_k + \mathbf{H}_k^T \mathbf{P}_k \mathbf{H}_k\right]^{-1}$$
$$\mathbf{K}_k = \mathbf{P}_k \mathbf{H}_k \mathbf{A}_k$$
$$\hat{\mathbf{w}}_{k+1} = \hat{\mathbf{w}}_k + \eta \mathbf{K}_k \mathbf{e}_k$$
$$\mathbf{P}_{k+1} = \mathbf{P}_k - \mathbf{K}_k \mathbf{H}_k^T \mathbf{P} k + \mathbf{Q}_k$$

with $\hat{\mathbf{w}}_{k+1} = \left[w_{i,k+1}, \ldots, w_{L,k+1}\right]^T$. Here, for the *k-th* time instant $\hat{\mathbf{w}}_k \in \mathbb{R}^L$ is the estimated weights vector such that $\hat{\mathbf{w}}_k$. $\mathbf{K}_k \in \mathbb{R}^{L \times n}$ is the Kalman gain, $\boldsymbol{e}_k \in \mathbb{R}^n$ is the difference between real output $\mathbf{y}_k \in \mathbb{R}^n$ and estimated output $\hat{\mathbf{y}}_k \in \mathbb{R}^n$, $\mathbf{P}_k \in \mathbb{R}^{L \times L}$ is the error covariance matrix. $\mathbf{R}_k \in \mathbb{R}^{n \times n}$ is the measurement error covariance matrix, $\mathbf{Q}_k \in \mathbb{R}^{L \times L}$ is the process noise covariance matrix, $\eta \in \mathbb{R}$ is a scalar learning rate parameter (typically set between 0.001 and 1), and $\mathbf{H}_k \in \mathbb{R}^{n \times L}$ contains partial derivatives of network states with respect to each *j-th* component $w_{j,k}$ of $\hat{\mathbf{w}}_k$ given by

$$\mathbf{H}_k = \left\{ \left[H_1, \ldots, H_n\right]^T \mid H_i = \frac{\partial \hat{x}_{i,k}}{\partial w_{j,k}} \right\} \tag{4}$$

where $i = 1, \ldots, n,$ and $j = 1, \ldots, L.$

3. HELICOPTER MODEL

The mathematical model for the 2-DOF Helicopter (Quanser) system can be determined from the following Lagrange formalism

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}\mathcal{L}(\mathbf{q},\dot{\mathbf{q}}) - \frac{\partial}{\partial q_i}\mathcal{L}(\mathbf{q},\dot{\mathbf{q}}) = Q_i \tag{5}$$
$$i = 1, \ldots, N$$

with

$$\mathcal{L}(\mathbf{q},\dot{\mathbf{q}}) = \mathcal{V}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{P}(\mathbf{q}),$$

where $\mathbf{q} = [q_1, \ldots, q_N]$ is the angular positions vector in generalized coordinates form, Q_i external generalized forces, y $\mathcal{L}$ the system's Lagrangian defined as the difference between kinetic $\mathcal{V}$ and potential $\mathcal{P}$ energy.

$$Q_i = \left[K_{pp}V_p + K_{py}V_y - B_p\dot{\theta}, K_{yy}V_y + K_{yp}V_p - B_y\dot{\phi}\right]$$
$$\mathcal{V} = \frac{1}{2}m_{he}\left(-sin(\phi)\dot{\phi}cos(\theta)l_{cm} - cos(\phi)sin(\theta)\dot{\theta}l_{cm}\right)^2$$
$$+\frac{1}{2}J_{eqp}\dot{\theta}^2 + \frac{1}{2}J_{eqy}\dot{\phi}^2 + \frac{1}{2}m_{he}cos^2(\theta)\dot{\theta}^2 l_{cm}^2$$
$$+\frac{1}{2}\left(-cos(\phi)\dot{\phi}cos(\theta)l_{cm} + sin(\theta)sin(\phi)\dot{\theta}l_{cm}\right)$$
$$\mathcal{P} = m_{he}g\ sin(\theta)l_{cm}$$

For the helicopter model, the previous expressions are obtained from the free-body diagram shown in Fig. 1 and Table 1, where the generalized coordinates vector is given by $\mathbf{q} = [\theta, \phi]$.

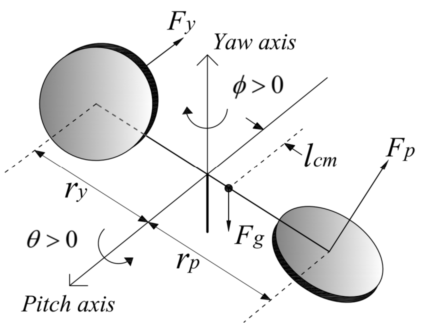

Fig. 1. Schematic of the helicopter

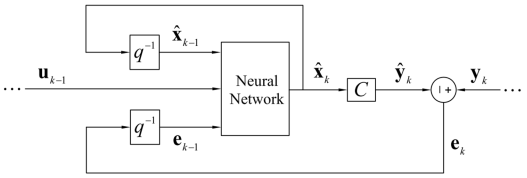

Fig. 2. The NNSSIF structure model

Table 1: System parameters

Symbol	Description	Value
J_{eqp}	Inertia about pitch	0.0384 $[[Kg.m^2]$
J_{eqy}	Inertia about yaw	0.00432 $[Kg.m^2]$
m_e	Total moving mass	1.3872 $[Kg]$
l_{cm}	Center of mass	0.186 $[m]$
B_p	V. damping – pitch	0.800 $[N/N]$
B_y	V. damping - yaw	0.318 $[N/N]$
K_{pp}	Force of pitch/pitch	0.204 $[N.m/V]$
K_{yy}	Force of yaw/yaw	0.072 $[N.m/V]$
K_{py}	Force on pitch/yaw	0.0068 $[N.m/V]$
K_{yp}	Force on yaw/pitch	0.0219 $[N.m/V]$
V_p	Peak voltage - pitch	± 24 $[V]$
V_y	Peak voltage - yaw	± 15 $[V]$
g	Force of gravity	9.81 $[m/s^2]$

In this way, solving (5), the matrix form that describes the system's dynamics can be determined as

$$D(\mathbf{q})\ddot{q}+C(\mathbf{q},\dot{\mathbf{q}})\dot{q}+F(\dot{\mathbf{q}})+G(\mathbf{q})=\mathcal{T} \qquad (6)$$

with

$$D=\begin{bmatrix} J_{eqp}+m_{he}l_{cm} & 0 \\ 0 & J_{eqy}+m_{he}l_{cm}\left(l_{cm}-sin^2(\theta)\right) \end{bmatrix}$$

$$C=\begin{bmatrix} 0 & m_{he}l_{cm}^2 sin(\theta)cos(\theta)\dot{\phi} \\ -2m_{he}l_{cm}sin(\theta)cos(\theta)\dot{\phi} & 0 \end{bmatrix}$$

$$F=\begin{bmatrix} B_p\dot{\theta} \\ B_y\dot{\phi} \end{bmatrix} \qquad G=\begin{bmatrix} 2m_{he}l_{cm}cos(\theta)g \\ 0 \end{bmatrix}$$

$$\mathcal{T}=\begin{bmatrix} K_{pp}V_p+K_{py}V_y \\ K_{yy}V_y+K_{yp}V_p \end{bmatrix}$$

where matrix $D(\mathbf{q})\in\mathbb{R}^{N\times N}$ includes terms related to inertia moments, $C(\mathbf{q},\dot{\mathbf{q}})\in\mathbb{R}^{N\times N}$ describes damping, $F(\dot{\mathbf{q}})\in\mathbb{R}^{N}$ is the stiffness matrix, $G(\mathbf{q})\in\mathbb{R}^{N}$ contains gravity force terms, and $\mathcal{T}\in\mathbb{R}^{N}$ is the thrust force vector. Thus, continuous-time, nonlinear model in affine form (7), is obtained from (6) defining the state vector

$$\mathbf{x}=\left[x_1,x_2,x_3,x_4\right]^T=\left[\theta,\phi,\dot{\theta},\dot{\phi}\right]^T,$$

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x})+\mathbf{h}(\mathbf{x})\mathbf{u}, \qquad (7)$$

considering

$$\mathbf{f}(\mathbf{x})=\begin{bmatrix} x_3 \\ x_4 \\ \dfrac{\Gamma_1(\mathbf{x})}{\Delta_1(\mathbf{x})}+\dfrac{\Gamma_2(\mathbf{x})}{\Delta_1(\mathbf{x})} \\ \dfrac{\Gamma_3(\mathbf{x})}{\Delta_2(\mathbf{x})}+\dfrac{\Gamma_4(\mathbf{x})}{\Delta_2(\mathbf{x})} \end{bmatrix}, \quad \mathbf{h}(\mathbf{x})=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{K_{pp}}{\Delta_1(\mathbf{x})} & \dfrac{K_{py}}{\Delta_1(\mathbf{x})} \\ \dfrac{K_{yp}}{\Delta_2(\mathbf{x})} & \dfrac{K_{yy}}{\Delta_2(\mathbf{x})} \end{bmatrix}$$

and $\mathbf{u}=\left[V_p,V_y\right]$, where

$$\begin{aligned}
\Gamma_1(\mathbf{x}) &= m_{he}(x_4)^2 sin(x_1)l_{cm}cos(x_1) \\
&\quad +m_{he}gcos(x_1)l_{cm} \\
\Gamma_2(\mathbf{x}) &= -B_p x_3 \\
\Gamma_3(\mathbf{x}) &= 2m_{he}x_4 sin(x_1)l_{cm}cos(x_1)x_3 \\
\Gamma_4(\mathbf{x}) &= -B_y x_4 \\
\Delta_1(\mathbf{x}) &= J_{eqp}+m_{he}l_{cm}^2 \\
\Delta_2(\mathbf{x}) &= J_{eqy}+m_{he}cos^2(x_1)l_{cm}^2.
\end{aligned}$$

3.1 Neural approximation of the model

From (1) and (7), in (8) a neural model for the helicopter is proposed, based on a neural network representing the state-space innovations form model (NNSSIF) shown in Fig. 2.

$$\begin{aligned}
\hat{x}_{1,k+1} &= w_{11,k}z_{1,k}+\Omega_{12}x_{3,k} \\
\hat{x}_{2,k+1} &= w_{21,k}z_{2,k}+\Omega_{22}x_{4,k} \\
\hat{x}_{3,k+1} &= w_{31,k}z_{3,k}+w_{32,k}z_{4,k}+w_{33,k}z_{5,k} \\
&\quad +\Omega_{34}u_{1,k}+\Omega_{35}u_{2,k} \\
\hat{x}_{4,k+1} &= w_{41,k}z_{6,k}+w_{42,k}z_{7,k}+w_{43,k}z_{8,k} \\
&\quad +\Omega_{44}u_{1,k}+\Omega_{45}u_{2,k}
\end{aligned} \qquad (8)$$

with

$$\hat{\mathbf{w}}_k=[w_{11,k},w_{21,k},w_{31,k},w_{32,k},w_{33,k},w_{41,k},w_{42,k},w_{43,k}]$$

$$\mathbf{z}_k=\begin{bmatrix} z_{1,k} \\ z_{2,k} \\ z_{3,k} \\ z_{4,k} \\ z_{5,k} \\ z_{6,k} \\ z_{7,k} \\ z_{8,k} \end{bmatrix}=\begin{bmatrix} s(x_{1,k}) \\ s(x_{2,k}) \\ s(x_{3,k}) \\ s^2(x_{1,k})s^2(x_{4,k}) \\ s(x_{1,k}) \\ s(x_{4,k}) \\ s^2(x_{1,k})s(x_{3,k})s(x_{4,k}) \\ s^2(x_{1,k})s(x_{4,k}) \end{bmatrix}, \qquad (9)$$

where $z_{i,k}$ with $i=1,\ldots,8$ are high-order terms according to (3), and the set of fixed weights established by $\{\Omega_{12},\Omega_{22},\Omega_{34},\Omega_{35},\Omega_{44},\Omega_{45}\}$, it consists of real constants different to zero to avoid losing controllability on the system. Further, the terms $\hat{x}_{i,k}$ with $i = 1, \dots, 4$, are the estimated states of the system presented in (7).

4. CONTROLLER DESIGN

Control scheme shown in Fig. 3 contains the structure used to solve the on-line control problem, where a NNSSIF is used to approximate the helicopter model; according to (2), the synaptic weights are adjusted each k instant by an extended Kalman filter (EKF), using the estimation error $\boldsymbol{e}_k$ of the system states to find optimal weights of network

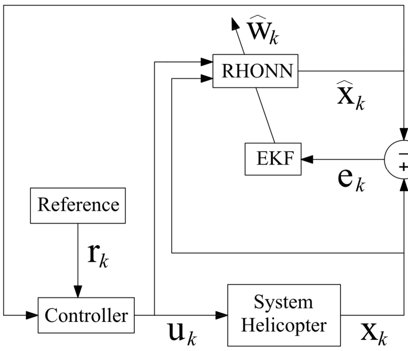

Fig. 3. Control scheme

4.1 Block control with a neural system identifier

Assuming that $x_{i,k} = \hat{x}_{i,k}$, from (8) the variable's blocks are defined as

$$\mathbf{x}_{1,k} = \begin{bmatrix} \hat{x}_{1,k} - x_{1r,k} \\ \hat{x}_{2,k} - x_{2r,k} \end{bmatrix} \qquad \mathbf{x}_{2,k} = \begin{bmatrix} \hat{x}_{3,k} \\ \hat{x}_{4,k} \end{bmatrix},$$

so that the system estimated by the network is expressed in the block controllable form, that is

$$\begin{aligned} \mathbf{x}_{1,k+1} &= \mathbf{f}_{1,k}(\mathbf{x}_{1,k}, \hat{\mathbf{w}}_k) + \mathbf{B}_1 \mathbf{x}_{2,k} \\ \mathbf{x}_{2,k+1} &= \mathbf{f}_{2,k}(\mathbf{x}_{1,k}, \mathbf{x}_{2,k}, \hat{\mathbf{w}}_k) + \mathbf{B}_2 \mathbf{u}_k \end{aligned} \tag{10}$$

with

$$\mathbf{f}_{1,k} = \begin{bmatrix} f_{1,k}(\mathbf{x}_{1,k}, \hat{\mathbf{w}}_k) \\ f_{2,k}(\mathbf{x}_{1,k}, \hat{\mathbf{w}}_k) \end{bmatrix} \tag{11}$$

$$f_{1,k} = w_{11,k} s(\hat{x}_{1,k}) - x_{1r,k+1}$$

$$f_{2,k} = w_{21,k} s(\hat{x}_{2,k}) - x_{2r,k+1}$$

$$\mathbf{B}_1 = \begin{bmatrix} \Omega_{12} & 0 \\ 0 & \Omega_{22} \end{bmatrix} \tag{12}$$

$$\mathbf{f}_{2,k} = \begin{bmatrix} f_{3,k}(\mathbf{x}_{1,k}, \mathbf{x}_{2,k}, \hat{\mathbf{w}}_k) \\ f_{4,k}(\mathbf{x}_{1,k}, \mathbf{x}_{2,k}, \hat{\mathbf{w}}_k) \end{bmatrix} \tag{13}$$

$$\begin{aligned} f_{3,k} &= w_{31,k} s(\hat{x}_{3,k}) + w_{32,k} s^2(\hat{x}_{1,k}) s^2(\hat{x}_{4,k}) \\ &\quad + w_{33,k} s(\hat{x}_{1,k}) \end{aligned}$$

$$\begin{aligned} f_{4,k} &= w_{41,k} s(\hat{x}_{4,k}) + w_{42,k} s^2(\hat{x}_{1,k}) s(\hat{x}_{3,k}) s(\hat{x}_{4,k}) \\ &\quad + w_{43,k} s^2(\hat{x}_{1,k}) s(\hat{x}_{4,k}) \end{aligned}$$

$$\mathbf{B}_2 = \begin{bmatrix} \Omega_{34} & \Omega_{35} \\ \Omega_{44} & \Omega_{45} \end{bmatrix} \tag{14}$$

where control matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ must have full rank, in order for the system to not lose controllability; particularly, in this case $rank(\mathbf{B}_1) = rank(\mathbf{B}_2) = 2$. Applying block control technique, the error vector corresponding to the first block is defined as

$$\mathbf{z}_{1,k} = \mathbf{x}_{1,k} = \begin{bmatrix} z_{1,k} \\ z_{2,k} \end{bmatrix}$$

taking one step ahead

$$\mathbf{z}_{1,k+1} = \mathbf{x}_{1,k+1} = \mathbf{f}_{1,k} + \mathbf{B}_1 \mathbf{x}_{2,k}.$$

Thus, assuming $x_{2,k}$ as a pseudo control required to drive the error $\mathbf{z}_{1,k}$ to zero, for the expression

$$\mathbf{z}_{1,k+1} = \mathbf{f}_{1,k} + \mathbf{B}_1 \mathbf{x}_{2,k} = \mathbf{K}_1 \mathbf{z}_{1,k} \tag{15}$$

the following pseudo control law is derived

$$\mathbf{x}_{2r,k} = [\mathbf{B}_1]^{-1} [-\mathbf{f}_{1,k} + \mathbf{K}_1 \mathbf{z}_{1,k}]$$

where $\mathbf{K}_1$ is a *Schur* diagonal matrix $\mathbf{K}_1 = diag\{k_1, k_2\}$ with the purpose of guaranteeing exponential convergence of $\mathbf{z}_{1,k}$ to zero. Therefore, to set $\mathbf{x}_{2,k} = \mathbf{x}_{2r,k}$ the second error vector is established as

$$\mathbf{z}_{2,k} = \begin{bmatrix} z_{3,k} \\ z_{4,k} \end{bmatrix} = \mathbf{x}_{2,k} - \mathbf{x}_{2r,k}. \tag{16}$$

Thus, replacing (16) in (15) one obtains

$$\mathbf{z}_{1,k+1} = \mathbf{K}_1 \mathbf{z}_{1,k} + \mathbf{B}_1 \mathbf{z}_{2,k} \tag{17}$$

and from (16) the error dynamic corresponds to

$$\mathbf{z}_{2r,k+1} = \mathbf{x}_{2,k+1} - \mathbf{x}_{2r,k+1} = \mathbf{f}_{2,k} + \mathbf{B}_2 \mathbf{u}_k - \mathbf{x}_{2r,k+1}$$

with

$$\mathbf{x}_{2r,k+1} = [\mathbf{B}_1]^{-1} [-\mathbf{f}_{1,k+1} + \mathbf{K}_1 \mathbf{z}_{1,k+1}],$$

where

$$\mathbf{f}_{1,k+1}(\mathbf{x}_{1,k+1}, \hat{\mathbf{w}}_{k+1}) = \begin{bmatrix} w_{11,k+1} s(\hat{x}_{1,k+1}) - x_{1r,k+2} \\ w_{21,k+1} s(\hat{x}_{2,k+1}) - x_{2r,k+2} \end{bmatrix}.$$

It should be mentioned, that the values of position reference signals $x_{1,k}$ and $x_{2,k}$ for the instants $k+1$ and $k+2$ were calculated by means of a discrete-time polynomial approximation using p previous steps according to (Abidi, 2007). In this way, a simplified form of the second block in new coordinates is obtained as

$$\mathbf{z}_{2,k+1} = \tilde{\mathbf{f}}_{2,k} + \mathbf{B}_2 \mathbf{u}_k \tag{18}$$

with $\tilde{\mathbf{f}}_{2,k} = \mathbf{f}_{2,k} - \mathbf{x}_{2r,k+1}$.

Finally, the block controllable form of the system is given by equations (17) and (18).

4.2 Discrete-time block inverse optimal control: Trajectory tracking

Let $\bar{\mathbf{z}}_{k+1}$ be a discrete-time nonlinear affine of block form system (17)-(18), and $\mathbf{y}_k$ an output

$$\bar{\mathbf{z}}_{k+1} = \bar{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + \bar{g}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\mathbf{u}_k \quad (19)$$

$$\mathbf{y}_k = h\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + J\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\mathbf{u}_k \quad (20)$$

with

$$\bar{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \begin{bmatrix} \mathbf{K}_1\mathbf{z}_{1,k} + \mathbf{B}_1\mathbf{z}_{1,k} \\ \tilde{\mathbf{f}}_{2,k} \end{bmatrix},$$

$$\bar{g}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \begin{bmatrix} 0_{2\times 2} \\ \mathbf{B}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \Omega_{34} & \Omega_{35} \\ \Omega_{44} & \Omega_{45} \end{bmatrix},$$

where $\bar{f}: \mathbb{R}^4 \to \mathbb{R}^2$ and $\bar{g}: \mathbb{R}^4 \to \mathbb{R}^{2\times 2}$ are smooth mappings. Assume that $\bar{f}(0) = 0$, $h(0) = 0$, and $rank(\bar{g}) = 4$.

System (19)-(20) is passive if exists a definite positive Lyapunov function $V(\mathbf{z}_{1,k}, \mathbf{z}_{2,k})$, such that

$$V\left(\mathbf{z}_{1,k+1}, \mathbf{z}_{2,k+1}\right) - V\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) \le \mathbf{y}_k^T\mathbf{u}_k \,. \quad (21)$$

Additionally, it's said that (19)-(20) is feedback passive if there exists a passivating law

$$\mathbf{u}_k = \boldsymbol{\alpha}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + \mathbf{v}_k, \qquad \boldsymbol{\alpha}, \mathbf{v}_k \in \mathbb{R}^2 \quad (22)$$

where $\boldsymbol{\alpha}$ is a smooth function, such that (19)-(20) can be described by

$$\bar{\mathbf{z}}_k = \tilde{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + \bar{g}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\mathbf{v}_k \quad (23)$$

$$\tilde{\mathbf{y}}_k = \tilde{h}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + J\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\mathbf{v}_k \quad (24)$$

with

$$\tilde{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \bar{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) + \bar{g}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\boldsymbol{\alpha},$$

$$\tilde{h}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \bar{g}^T\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) P \tilde{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right),$$

where $\mathbf{v}_k$ is the new input that satisfies (21), $\tilde{h}: \mathbb{R}^4 \to \mathbb{R}^2$ is a smooth mapping, and P is a symmetric and definite positive matrix.

Consider the discrete-time nonlinear system (19) with input (22) and output (24) be zero-state detectable, such that (21) is satisfied with the discrete-time Lyapunov function given by

$$V\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \frac{1}{2}\begin{bmatrix} \mathbf{z}_{1,k} \\ \mathbf{z}_{2,k} \end{bmatrix}^T P \begin{bmatrix} \mathbf{z}_{1,k} \\ \mathbf{z}_{2,k} \end{bmatrix}, \quad (25)$$

where $P = P^T > 0$, $P \in \mathbb{R}^4$ is established as

$$P = \begin{bmatrix} 10^5 & 0.2\times10^5 & 0.5\times10^5 & 0.1\times10^5 \\ 0.2\times10^5 & 10^5 & 0.2\times10^5 & 0.5\times10^5 \\ 0.5\times10^5 & 0.2\times10^5 & 10^5 & 0.2\times10^5 \\ 0.1\times10^5 & 0.5\times10^5 & 0.2\times10^5 & 10^5 \end{bmatrix}.$$

Further, consider the following cost functional associated with system (19)

$$\mathcal{J} = \sum_{k=0}^{\infty} L\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}, \mathbf{u}_k^*\right) \quad (26)$$

where $L(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}, \mathbf{u}_k^*) = l(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}) + \mathbf{u}_k^{*T}\mathbf{u}_k^*$, $\mathbf{u}_k^*$ is the control law that minimizes (26), and $l(\mathbf{z}_{1,k}, \mathbf{z}_{2,k})$ is a positive semidefinite function. Therefore, from (24) the system (19) is globally asymptotically stable at the equilibrium point $\bar{\mathbf{z}}_k = 0$ doing the output feedback $\mathbf{v}_k = -\bar{\mathbf{y}}_k$. In this way, the optimal law that minimizes (26) and satisfies (21) assuming (25), is given by

$$\mathbf{u}_k^* = \boldsymbol{\alpha} = \left(I + J\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)\right)^{-1} h\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) \quad (27)$$

where

$$J\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \frac{1}{2}\bar{g}^T\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) P \,\bar{g}^T\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right)$$

$$h\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) = \bar{g}^T\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right) P \bar{f}\left(\mathbf{z}_{1,k}, \mathbf{z}_{2,k}\right),$$

and $I \in \mathbb{R}^{2\times 2}$ is the identity matrix.

5. RESULTS

The parameters used for the simulation and implementation correspond with a real Quanser prototype (Table 1). The control strategy developed in section 4, was evaluated considering its capability of tracking a circular trajectory with the nose of the helicopter. So, the desired trajectory in simulation for pitch and yaw corresponds to $x_{1r,k} = 10sin(2\pi kT_s/N)\,[deg]$ and $x_{2r,k} = 10cos(2\pi kT_s/N)[deg]$, where the period for the reference signals is $N = 4\,[s]$ for simulation and $N = 32\,[s]$ in implementation, maintaining initial resting conditions of the real plant given by (28). In both cases the sampling time was established as $T_s = 0.002\,[s]$. The results obtained are shown below where from Fig. 4 to Fig. 7 corresponds to simulation in MATLAB-SIMULINK using the solve ODE1 with fixed-step equal to T_s, while from the Fig. 8 to Fig. 13 the results are shown in real-time.

$$\mathbf{x}_{1,0} = \begin{bmatrix} -40[deg] \\ 0[deg] \end{bmatrix} \qquad \mathbf{x}_{2,0} = \begin{bmatrix} 0[deg/s] \\ 0[deg/s] \end{bmatrix} \quad (28)$$

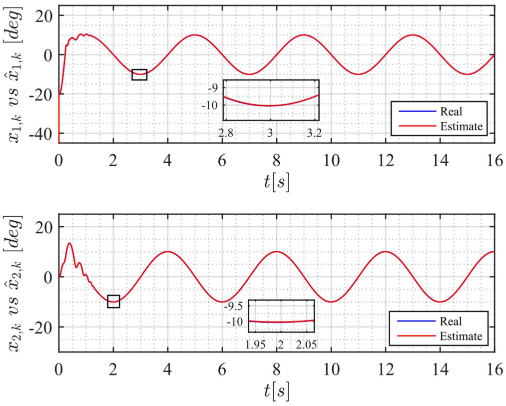

Fig. 4. Positions identification - Simulation

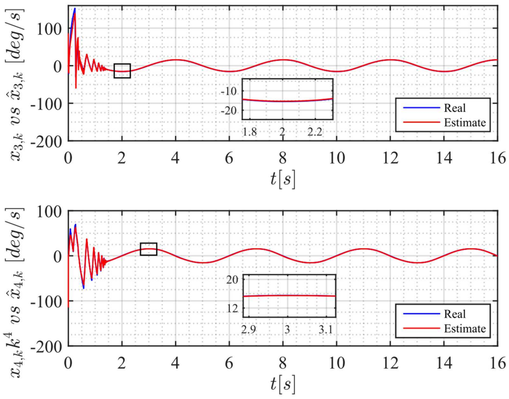

Fig. 5. Velocities identification - Simulation

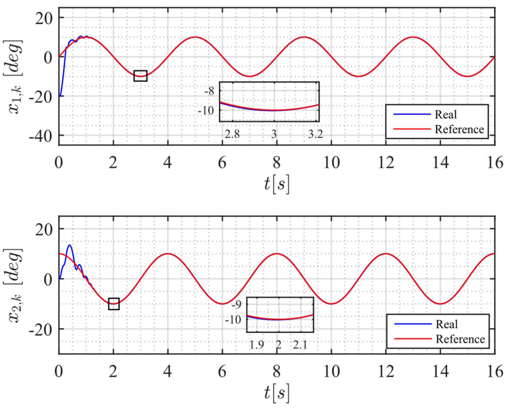

Fig. 6. Position tracking - Simulation

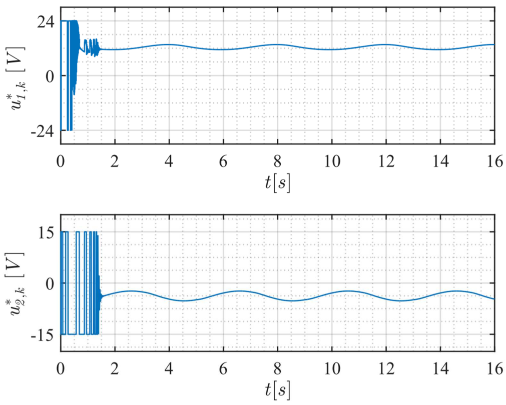

Fig. 7. Control law optimal - Simulation

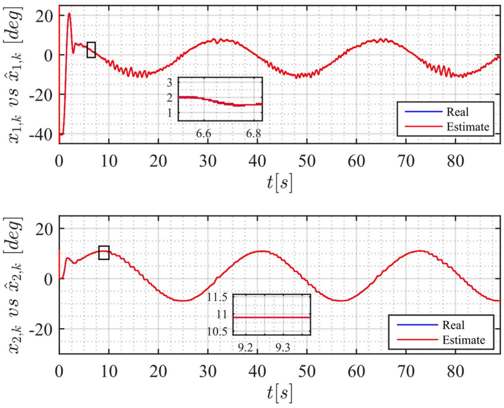

Fig. 8. Real-time positions identification

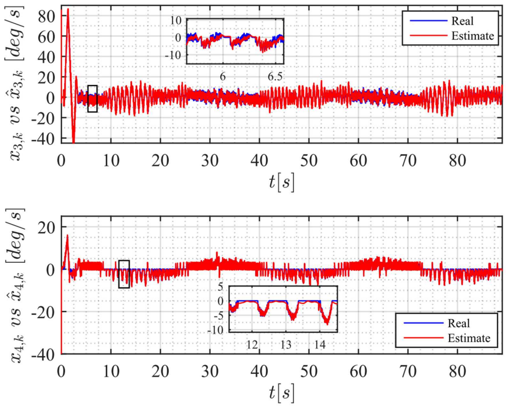

Fig. 9. Real-time velocities identification

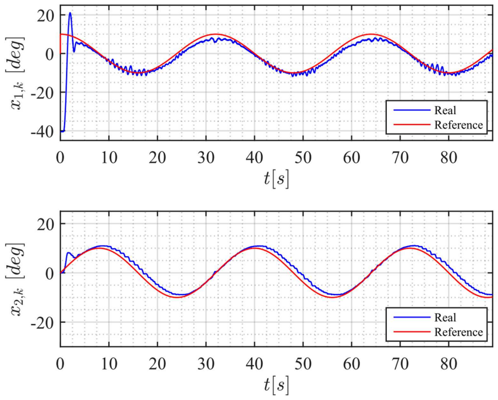

Fig. 10. Real-time position tracking

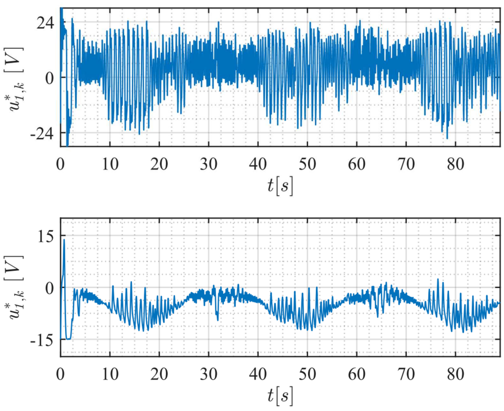

Fig. 11. Real-time control law optimal

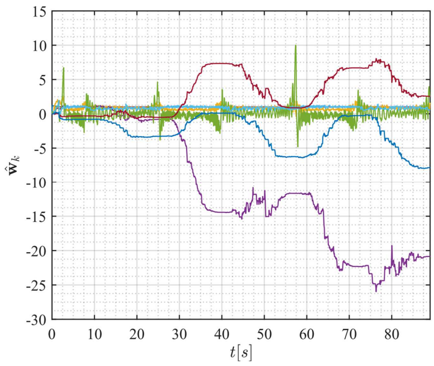

Fig.12. Real-time network weights

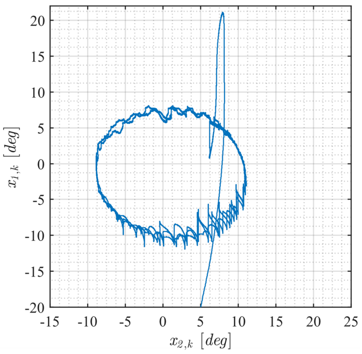

Fig. 13. Real-time phase portrait

6. CONCLUSIONS

From the simulation and implementation results, the outstanding capacity of RHONN to identify nonlinear systems with parameter variations is highlighted, using an EKF with synaptic weights adapted on-line and fixed weights on the diagonal of the control matrix of each block. Furthermore, the reduced order of the system given by the block controllable form and its use along with inverse optimal control, have shown acceptable results when applied to tracking of trajectories by means of smooth control laws.

REFERENCES

Abidi, K., Xu, J. X. and Xinghuo, Y. (2007). “On the discrete-time integral sliding-mode control”. IEEE Transactions on Automatic Control, Vol. 52, No. 4, pp. 709-715.

Boukadida, W., Benamor, A., Messaoud, H. and Siarry, P. (2019). “Multi-objective design of optimal higher order sliding mode control for robust tracking of 2-DoF helicopter system based on metaheuristics”. Aerospace Science and Technology.

Drakunov, S. V., Izosimov, D. B., Luk'yanov, A. G., Utkin, V. A. E. and Utkin, V. I. (1990). “The block control principle. I. Avtomatika i Telemekhanika”, Vol. 5, pp. 38-47.

Haykin, S. (2004). *Kalman filtering and neural networks*, Vol. 4, John Wiley & Sons.

Hernandez-Gonzalez, M., Alanis, A. Y. and Hernandez-Vargas, E. A. (2012). “Decentralized discrete-time neural control for a Quanser 2-DOF helicopter”. Applied Soft Computing, Vol. 12, No. 8, pp. 2462-2469.

Kosmatopoulos, E. B., Polycarpou, M. M., Christodoulou, M. A. and Ioannou, P. A. (1995). “High-order neural network structures for identification of dynamical systems”. IEEE Transactions on Neural Networks, Vol. 6, No. 2, pp. 422-431.

Kumar, E. V., Raaja, G. S. and Jerome, J. (2016). “Adaptive PSO for optimal LQR tracking control of 2 DoF laboratory helicopter”. Applied Soft Computing, Vol. 41, pp. 77-90.

Ornelas-Tellez, F., Sanchez, E. N. and Loukianov, A. G. (2012). “Discrete-time neural inverse optimal control for nonlinear systems via passivation”. IEEE Transactions on Neural Networks and Learning Systems, Vol. 23, No. 8, pp. 1327-1339.

Puskorius, G. V. and Feldkamp, L. A. (1994). “Neurocontrol of nonlinear dynamical systems with Kalman filter trained recurrent networks”. IEEE Transactions on Neural Networks, Vol. 5, No. 2, pp. 279-297.

Rojas-Cubides, H., Cortés-Romero, J., Coral-Enriquez, H. and Rojas-Cubides, H. (2019). “Sliding mode control assisted by GPI observers for tracking tasks of a nonlinear multivariable Twin-Rotor aerodynamical system”. Control Engineering Practice, Vol. 88, pp. 1-15.

Rovithakis, G. A. and Christodoulou, M. A. (2012). *Adaptive control with recurrent high-order neural networks: theory and industrial applications*. Springer Science & Business Media.

Sanchez, E. N., Alanís, A. Y. and Loukianov, A. G. (2008). *Discrete-time high order neural control*. Warsaw: Springer.

Simon, D. (2002). “Training radial basis neural networks with the extended Kalman filter”. Neurocomputing, Vol. 48, No. 1, pp. 455-475.

Steinbusch, A. and Reyhanoglu, M. (2019). “Robust Nonlinear Tracking Control of a 2-DoF Helicopter System”. In 2019 12th Asian Control Conference (ASCC), pp. 1649-1654. IEEE.