Optimal Power Dispatch of Distributed Generators in Direct-Current Networks with Nonlinear Loads

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Abstract.
In this paper has oriented the problem of optimal power flow problem in direct current grids with constant power loads and distributed generators. The optimal power flow problem is formulated as a nonlinear optimization problem and solved by using a master-slave algorithm. The master stage is responsible for dispatching all the distributed generators by employing the Vortex Search algorithm. In the slave stage, an efficient power flow method based on Taylor series named extended linearization method is used to calculate all the voltage variables and evaluating the objective function of the problem, which corresponds to the power loss minimization. A direct current distribution network composed of 21 nodes is employed as test case by comparing the numerical performance of the proposed approach with the nonlinear optimization packages available in GAMS. All the simulations are conducted via MATLAB software 2017*b* licensed by Universidad Tecnológica de Bolívar.

1. Introduction

Nowadays, direct current (DC) microgrids are a powerful electrical network tendency that becomes a possible economic and reliable alternative for providing electrical services to millions of users around the world [1]. The main advantage of using DC grids lie in the possibility of integrating multiple distributed energy resources, such as energy storage systems [2] (batteries, supercapacitors or fuel-cells) and renewable generation [3] (solar, wind technologies, among others) with DC-DC converters or ac-DC inverters, thus reducing the back-to-back topologies required to integrate them into conventional AC power grids [4]. Those reductions must be reflected in the total cost of the distributed energy resources, as well as lower power loss and high electrical efficiency of DC networks in comparison with their AC counterparts [5]. Another important driver of the popularization of DC grids is that they are easier to analyze and operate because disappear frequency and reactive power control, key concepts of classical AC grids [5].

One of the most important problems in DC grids is the optimal power flow problem (OPF) because it represents an essential tool for planning, operating, and controlling DC networks [5–7]. The OPF problem is interesting for the research community because it is nonlinear and non-convex, and solving it requires of purposing new and efficient methodologies [8, 9].

In the case of Colombian power system, regulatory entities have impulsed the massive integration of renewable energy sources and energy storage systems in the voltage levels; in addition, the concept of microgrids for isolated applications has also impulsed by utilities for improving their grids in terms of voltage profiles and power loss. These characteristics have converted DC power systems into a powerful alternative for improving the efficiency and reliability of the power system in Colombia, and this paper tries to contribute with powerful tools for its analysis as is the case of the OPF solvers.

According to the review of the state-of-the-art above, only two combinatorial optimization methods have been applied to solve OPF problems in DC power grids, such as the continuous genetic algorithm proposed in [1] and the black hole optimizer presented in [10]. In that sense, this paper identifies a research gap in the field and offers a VSA approach [11]. The main advantage of the proposed approach in this work, in comparison with convex methods, lies in the fact that the number of variables of the power flow problem remains constant and no eigenvalue decomposition is required to recover power flow variables [8].

To complement the proposed VSA algorithm in the master stage, a Taylor-based method recently proposed in [12] for power solution at the slave stage. This combination produces a new hybrid optimization approach ahead named VSA-TBM. Additionally, this method does not require the usage of specialized software, and it can be implemented in any programming language since its solution methodology is pure-algorithmic [1, 10].

The remainder of this paper is organized as follows: Section 2 presents the conventional OPF problem for DC power grids with a focus on the possibility of including distributed generation in the grid by controlling their percentages of penetration. Section 3 introduces the main characteristics of the VSA method as well as its evolution process and its application to the master problem; in addition, it shows the conventional TBM formulation for solving power flow problems in DC power grids with its use to the slave problem. Section 4 describes the test system, the numerical results, as well as the comparative methods. Finally, Section 5 draws the main conclusions and possible future research derived from this work, followed by acknowledgments and references.

2. Mathematical model

The optimal power flow problem in networks correspond to a nonlinear non-convex optimization problem, which has the power loss minimization as a usually objective function, subject to power balance, voltage regulation and elements capability constraints, these have hyperbolic restrictions [8, 9]. The complete mathematical model of the OPF problem is presented below

Objective function

$$\min p_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij} v_i v_j, \tag{1}$$

where p_{loss} is the objective function value associated to the power loss in all the resistive effects in the conductors of the DC grid, g_{ij} is the ij^{th} component of the conductance matrix, and v_i and v_j are the voltage values at nodes i and j, respectively. Note that n corresponds to the total number of nodes in the DC grid.

Set of constraints:

$$p_i^s = v_i \sum_{j=1}^{n} g_{ij} v_i v_j, \ \forall i \in \mathcal{S}, \tag{2}$$

$$p_i^{dg} - p_i^d = v_i \sum_{j=1}^{n} g_{ij} v_i v_j, \ \forall i \in \mathcal{N} - \mathcal{S}, \tag{3}$$

$$p_i^s \geq 0, \ \forall j \in \mathcal{S}, \tag{4}$$

$$v_i = v_{is}, \ \forall i \in \mathcal{S}, \tag{5}$$

$$0 \leq p_i^{dg} \leq p_i^{dg,\max}, \ \forall i \in \mathcal{N} - \mathcal{S}, \tag{6}$$

$$v_i^{\min} \leq v_i \leq v_i^{\max}, \ \forall i \in \mathcal{N} - \mathcal{S}, \tag{7}$$

where p_i^s represents all the power generation in the slack nodes (voltage controlled nodes, contained in the set $\mathcal{S}$), p_i^{dg} and p_i^d are the power generation in the distributed generation (DG's) and constant power consumption in all the demand nodes in the grid (contained in the set $\mathcal{N}-\mathcal{S}$), v_{is} are the output voltage in all the slack nodes, which are constant values; $p_i^{dg,\max}$ is the maximum power generation of a DG located at node i; and $v_i^{\min}$ and $v_i^{\max}$ are the minimum and maximum voltage regulation bounds allowed for the DC grid operation.

The complete interpretation of the mathematical model given from Equations (1) to (7) is described as follows: in Equation (1) is defined the objective function of the OPF problem related with the total power loss minimization; in Equations (2) and (3) are formulated the power balance constraints in all nodes of the DC grid, i.e., slack and constant power load nodes; Equations (4) and (5) show the positiveness characteristic of the power generation as well as the constant voltage performance in all the slack nodes, respectively; finally, in Equations (6) and (7) are presented the minimum and maximum capabilities of the DG's as well as the voltage regulation bounds in all the demand nodes of the network, respectively.

Due to the non-convexity of the optimal power flow problem, there are two main ways of solving this problem efficiently: *i)* using a convex reformulation via sequential quadratic approximations [8, 13], or *ii)* using metaheuristic approaches as reported in [1, 10]. Here, we adopted the second option for solving the OPF problem as presented in the next section.

3. Proposed methodology

To solve the OPF problem in this paper, we propose a master-slave methodology formed by the SCA in conjunction with the successive approximation power flow method.

3.1. Master stage

To solve the OPF problem in DC grids, we proposed a master strategy based on VSA, which is entrusted of determining the optimal power output in all the distributed generators. Algorithm 1 presents the pseudocode of the proposed VSA in the solution of the OPF problem in DC networks with penetration of distributed generation.

Data: Read data of the network and adjust parameters of the VSA
Define the initial center of the hypersphere μ_m;
Define the initial radius of the hypersphere r_m;
Generate the initial population S^m;
Verify the lower and upper bounds of all the individuals.;
Evaluate all the individuals s_i^m and find s_{best}^m;
for $m = 1 : m_{\max}$ **do**
    Update the center of the hypersphere μ_{m+1};
    Calculate the new radius of the hypersphere r_{m+1};
    Calculate the new popilation S^{m+1};
    Verify the lower and upper bounds of all the individuals.;
    Evaluate all the individuals s_i^{m+1} and find s_{best}^{m+1};
    Evaluate the number of non-consecutive improvements of z_f;
    if $k \geq k_{\max}$ **then**
        Select as solution of the problem μ_{m+1};
        Return the solution of the OPF problem;
        break;
    end
end
Result: Return the solution of the OPF problem

Algorithm 1: Proposed optimization methodology based on the VSA for OPF analysis

For a complete interpretation of the mathematical symbols in Algorithm 1, the following references can be consulted [11, 14].

3.2. Slave stage

Power flow analysis for DC grids can be analyzed with different methodologies, including Gauss-Seidel, successive approximations, Newton-Raphson, and Taylor-based methods (TBMs), among others. Here, we select a TBM recently reported in [12]. This approach works by an iterative procedure based on a simplified Jacobian rule as presented in the recursive formula defined by Equation (8).

$$v_d^{t+1} = \left[G_{dd} + \mathbf{diag}^{-2}(v_d^t)\mathbf{diag}\left(p^{dg} - p^d\right)\right]^{-1}\left[2\mathbf{diag}^{-1}(v_d^t)\left[p^{dg} - p^d\right] - G_{ds}v_s\right], \quad (8)$$

where p^{dg} and p^d are vectors that contain all the power generation in the DGs and the constant power consumptions in all the demand nodes; v_d is the vector that contains all the unknown voltage profiles, v_s is the vector that contains all the slack voltages (i.e., known voltage profiles); and G_{ds} and G_{dd} are sub-matrices of the conductance matrix that relates generation and demand nodes. Note that $\mathbf{diag}(v_d)$ generates a diagonal matrix with the components of the vector v_d. In addition, t is incremented until $t_{\max}$ (maximum number of iterations) and the search stop when $\max\left\{\left|v_d^{t+1} - v_d^t\right|\right\} \leq \epsilon$. It is important to mention that ϵ represents the maximum convergence error allowed between two consecutive iterations. In specialized literature, this parameter is typically selected as 1×10^{-10} [12].

4. Numerical validation

The 21-node test feeder used in the specialized literature is employed [8] for validating the proposed master-slave optimization approach. This test system consists of 21 nodes and 20 lines with multiple constant power loads. In addition, said system includes one ideal voltage generator at node 1 (i.e., voltage-controlled node). All of the information of this test system can be consulted in [8]. This study analyzes the possibility of installing from three DGs considering penetration percentages from 20% to 60% of the total power consumption. Note that, via heuristic search methods, nodes 9, 12 and 16 were selected for DGs' location [10].

Table 1 shows the generation at each DG considering different percentages of power penetration, total power generation, and objective function values that are total power losses in the network DC. Note that, in each simulation case, the maximum power injection allowed into the DC grid reaches 1,0954 p.u, 2.2160 p.u and 3.3240 p.u for distributed generator penetration scenarios of 20%, 40% and 60%, respectively. In that sense, from the fifth column in Table 1, it can be noticed that the proposed VSA-TBM method as well as the GAMS optimizer use more than 99% of the maximum generation allowed to reduce total power losses in the DC network. In terms of objective function minimization, note that the method proposed in this work presents an estimation error lower than 0.30% (compared to the SCIP solver) for each simulation case. Therefore, such hybrid VSA-TBM method offers adequate numerical convergence compared to large-scale nonlinear packages.

Table 1: Numerical comparison between the proposed approach and the GAMS package [15]

Penetration [%]	DG 9 [p.u]	DG 12 [p.u]	DG 16 [p.u]	Total Gen [p.u]	Losses [p.u]	Reduction [%]
Solutions provided by the VSA–TBM method proposed in this work						
20%	0.0168	0.0405	1.0381	1.0954	0.13823	49.91
40%	0.2472	0.6981	1.2707	2.2160	0.06611	76.04
60%	0.8451	1.0294	1.4495	3.3234	0.03061	88.90
Solutions provided by the GAMS and the SCIP solver						
20%	3×10^{-6}	0.1426	0.9654	1.1080	0.13662	50.50
40%	0.2464	0.6970	1.2726	2.2160	0.06621	76.00
60%	0.8442	1.0254	1.4544	3.2400	0.03071	88.87

It is important to stand out that the power delivered by each distributed generator exhibits small variations compared to GAMS optimizer results in the solutions obtained by means of the proposed approach. These differences can be attributed to the nonlinear continuous nature of the OPF problem and to the use of metaheuristic method, which implies that multiple distributed generator power combinations with identical objective functions could exist. The proposed VSA-TBM method and the comparative method via GAMS implementation exhibit similar performance in terms of power losses reduction, which clearly validates the proposed approach in terms of numerical convergence. In addition, when the reduction of the final power loss respect to the case without distributed generation are compared as can be seen in the seventh column of Table 1, we can be sure that our proposed approach is efficient and reliable for OPF analysis in relation to well-known commercial solvers.

5. Conclusion and future work

This paper presented a combination of the vortex search algorithm approach in conjunction with the Taylor series-based method for solving the OPF problem in DC power grids. The VSA method is a soft variant of the widely-known random search and pattern search algorithms, which offer an adequate trade-off between exploration and exploitation of the solution space for continuous optimization models as the case of the OPF problem. To confirm this advantage, the large-scale nonlinear optimization package SCIP solver (available for GAMS) was used as comparative method, which allowed observing the excellent numerical convergence of the proposed hybrid VSA-TBM method for OPF analysis.

As future works, the VSA-TBM method presented in this paper could be used for solving the problem of optimal location and sizing of distributed generation in DC power grids in conjunction with binary metaheuristic techniques or use it within an economic dispatch strategy of DC networks.

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