Dynamic analysis of three disc brakes in motorcycles

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Abstract.
Introduction: The braking system in motorcycles is of vital importance, taking into account that its operation is based on the friction between two surfaces in contact generating heat and therefore be compromised the brake fluid, thermo elastic deformation in the contact surface, degradation and failure of the material, to which the safety of the occupants can be attributed. **Objective:** Determine mathematically dynamic calculations of the braking system to improve the aerodynamic conditions of the brakes for heat loss. **Materials and Methods:** With the help of SolidWorks software, the geometrical model of three brake discs of different cylinders was carried out in order to identify the elements subjected to maximum dynamic conditions in relation to the geometry of the brake disc. **Results and Discussion:** The results obtained show that with the mathematical calculations it was possible to validate the correct functioning of the braking system at different operating conditions, that the systems that have greater cylinder capacity guarantee better braking distance for a given time and speed. **Conclusions:** These systems work in optimal conditions always guaranteeing high levels of safety and operation compared to other types of geometries, moreover to being able to determine their operating conditions in different working conditions, into account the mathematical dynamic calculations carried out.
Keywords: Dynamic, Friction, Solidworks, Disc Brakes, Motorcycle.

NOMENCLATURE:

M_p = Pilot mass. (Kg)
M_m = Motorcicle mass. (Kg)
$P_{front\ wheel}$ = Pressure on front wheel. (Pa)
R_b = Thermal transmission. ($J/s \times m^2 \times °C$)
T_F = Total force to be exerted by the pistons of the clamp on the brake pads. (N)
T_m = Total mass (Kg)
V_F = Final speed of the vehicle = 22.22 (m/s)
V_i = Initial speed of the vehicle. (m/s)
W_F = Front wheel weight. (Kg)
d_{axis} = Distance between centers. (mm)
$d_{cgd\ m}$ = Distance of center of gravity of motorcycle (mm)
$d_{cgd\ p}$ = Distance of center of gravity of pilot (mm)
$h_{cdg\ m}$ = Height of the center of gravity of motorcycle. (mm)
$h_{cdg\ p}$ = Height of the center of gravity of pilot. (mm)
D = Braking distance. (m)
F = Braking force. (N)
F = Braking force. (N)
a = Aceleration (m/s)
g = Gravity (m/s^2)
t = Braking time. (s)
μ = Coefficient of pavement. (Adimensionless)

1. Introduction

During the first half of the 20th century all motorcycles used drum brakes. These brakes were perfectly valid, but they warmed up easily, they were a bit heavy and changing the shape of their shoes was complicated. In the mid-70s began to see disc brakes on the high end motorcycles. They were only mounted on the front wheel and their price was high, but little by little, due to their better behavior, they became generalized as in the case of automobiles. Nowadays all medium-high range motorcycles have disc brakes on both wheels and sometimes the front wheel disc is double. Some mid-range bikes retain the drum brake on the rear wheel and only the simplest motorcycles have drums on the front axle, which is the case of some scooter models that we can find on the market today (Criado, 2012).

The braking consists of a deceleration phase of the movement of the motorcycle; during braking, the heat of friction is generated in a dispersion optimally to avoid the accumulation of heat that can lead to vaporization of the brake fluid, thermo-elastic deformation at the contact surface, degradation and different types of faults. the contact surface (material) (García-León, 2014). In addition, this heat must be dissipated quickly to the medium to avoid the reduction of the coefficient of friction between the brake and the brake pad (Talati and Jalalifar, 2009)(Todeschini *et al.*, 2014). Together, the discs suffer mechanical stress all the time, so that wear and deterioration of the structure is generated, to which an inadequate maintenance is added causing the component failure quickly (García-León and Flórez-Solano, 2017)(Dhaubhadel, 1996).

In other investigations, an analysis was carried out that allowed to identify the physical and mechanical characteristics (such as the type of material, geometry, thermal properties, friction and wear) and in this way to be able to compare the data obtained with those found in the bibliography. In addition to determining the service life of the brakes under the conditions to which it is subjected (García-León, 2017)(García-León and Perez Rojas, 2017). These data are very important for the validation of the correct functioning of braking systems that are of great importance for the automotive industry.

(Zurin, Talib and Ismail, 2017) they performed a CFD analysis on solid and ventilated motorcycle brakes, in order to validate the thermal behavior during braking in terms of temperature distribution in the disc geometry. The results showed that the ventilated disc brake had a better braking performance in terms of heat transfer compared to the solid disc.

Therefore, it is important to carry out studies using finite elements in order to optimize the behavior of these braking systems and be able to analyze the phenomenon that occurs in different operating conditions as in the investigations carried out by (Manohar Reddy, Mallikarjuna and Ganesan, 2006). The numerical methods provide solutions for discrete points within the limits of the problem and offer an approximation of the exact solution. However, when dealing with the solution for a finite number of discrete points, the method is simplified, for now solving a system of simultaneous algebraic equations, instead of the differential equation. Being the solution of simultaneous equations, the ideal task for computers. This occurs through the discretization of the domains in question, the discretization being the replacement of differential equations in a system of algebraic equations (García-León, Flórez-Solano and Rodriguez-Castilla, 2019).

Finally, the term sustainable development (SD) appears with the idea of efficiently and rationally managing the resources available to society, in such a way that it is possible to improve the well-being of the current population without compromising the quality of life of future generations. It is also considered an emerging trend that many companies see as a challenge that can affect their competitiveness by not perceiving its strategic character. Achieving economic growth that preserves and enriches rather than destroying the natural sources on which human activity is inevitably based is the main objective of any sustainability strategy (Carro Suárez *et al.*, 2017).

On the other hand, sustainable development in the automotive sector is no exception, currently based on the trend experienced in Sustainable Consumption and Production, taking into account the terms of energy efficiency and the elements that suffer most from wear in the vehicle which are the disc brakes and the engine. In this way, our current mobility system impacts on the quality of life through pollutant emissions resulting from the poor design of automotive elements (Conraud Koellner and Arredondo Hidalgo, 2018).

For this research, the dynamic analysis will be obtained by obtaining the geometries of the brakes with the help of the SolidWorks design software to study the behavior of the temperature on the friction surface of the discs.

2. Materials and method

The kinetic and potential energy of the vehicle is quickly transformed into thermal energy by the brakes. In order for the braking system to function properly, the heat generated must dissipate quickly as possible so that the brakes do not overheat and thereby compromise the performance and safety of the system. The movement of the vehicle allows the dissipation of heat by convection and radiation. When intense or repetitive braking occurs it causes the temperature to rise to a certain limit, known as the saturation temperature, which depends on the thermal dissipation capacity of the brake disc, which is influenced by the type of geometry and type of brake material.

On the other hand, the following figure 1 shown the methodology applied to the analysis for the brake systems of the motorcycles:

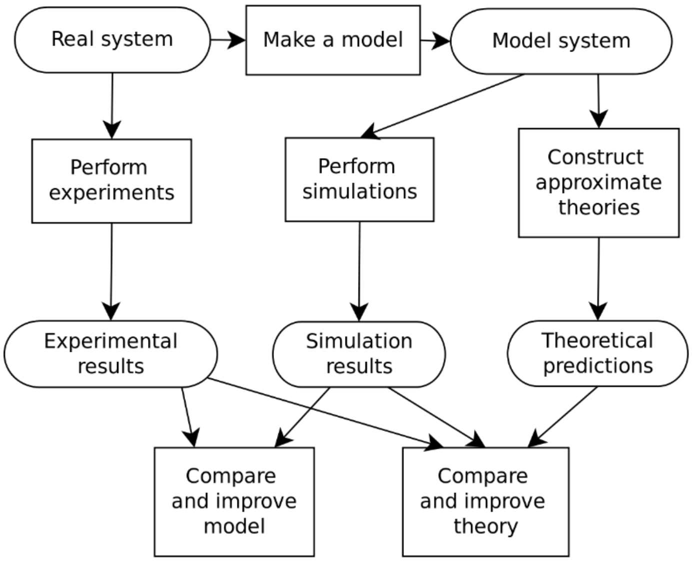

Figure 1. Methodology applied to the results.

For the carried out the calculations, we made theoretical predictions about the dynamic behavior in motorcycles according to disc brake parameters currently used in different types of motorcycles. The results were compared with bibliographic sources where they report some conditions of the tests performed, to obtain a broader view of the behavior of braking systems. The above helps to improve the mechanical properties of the material of experimentally from for the conditions to which they are exposed and thus contribute to sustainable development based on the waste and safety of the braking system.
In the analysis carried out, the saturation limit was not included, given that ideal braking conditions are assumed, such as: maximum speed of 80 km/h and an environment with a temperature of 25°C. Due to the above, the physical and thermal properties of the discs were obtained, in addition the calculations are developed for the disc brake of the motorcycle Suzuki Best 125cc, AKT CPI 135cc and Suzuki GS 150cc. The use of Solidworks design software was performed only to show the geometries used on the calculus of the three-disc brakes were performed, as shown in Figures 2 to 4:

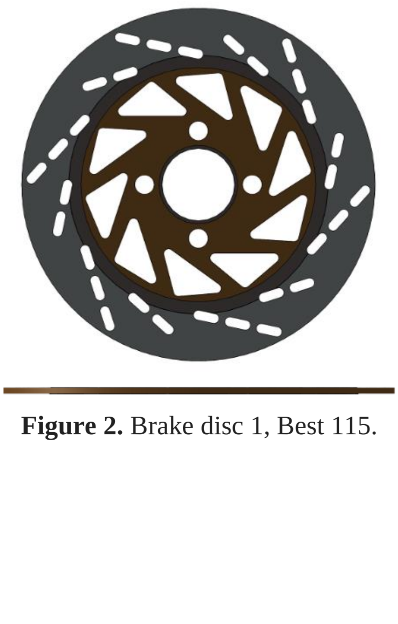
Figure 2. Brake disc 1, Best 115.

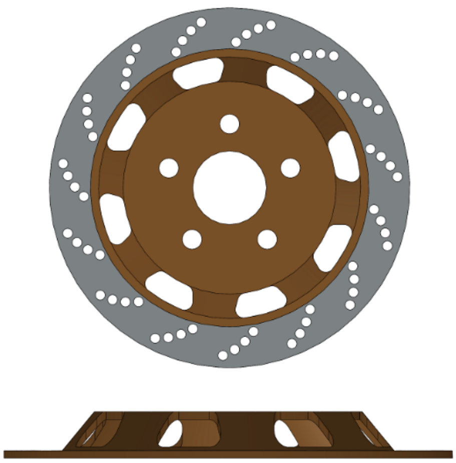

Figure 3. Brake disc 1 - CPI 135.

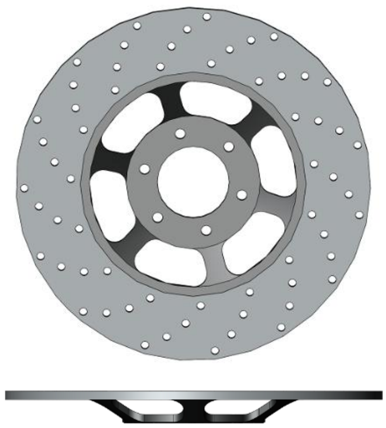

Figure 4. Brake disc 3 - GS 150.

The physical and thermal properties of this material were determined taking into account the references of Cengel and Riley (Pan *et al.*, 2015) (Cengel, 2007).

Table 1. Physical properties of AISI 310 stainless steel.

DENSITY	**Lbm/in^3**	**gr/cm^3**
68°F (20°C)	0.29	8.03
COEFFICIENT OF THERMAL EXPANSION	**(µin/in)*°F**	**(µm/m)*K**
68°F-212°F (20°C-100°C)	8.8	15.9
68°F-932°F (20°C-500°C)	9.5	17.1
68°F-1832°F (20°C-1000°C)	10.5	18.9
THERMAL CONDUCTIVITY	**Btu/hr*ft*F**	**W/m*K**
68°F-212°F (20°C-100°C)	8	13.8
68°F-932°F (20°C-500°C)	10.8	18.7
ELASTICITY MODULE	**psi**	**Gpa**
In tension (E)	2.90E+07	200
In shear (G)	1.12E+07	77
SPECIFIC HEAT	**Btu/lbm*°F**	**J/kg*K**
32°F-212°F (0°C-100°C)	0.12	502

The brake disc of a motorcycle is made of stainless steel commonly of the 300 series (AISI 310) (García-León, Flórez-Solano and Acevedo-Peñaloza, 2018). Figure 5, it will show the general properties, the chemical composition, the mechanical properties, the resistance to corrosion, the resistance to oxidation at high temperatures and the thermal treatment present in this type of steel.

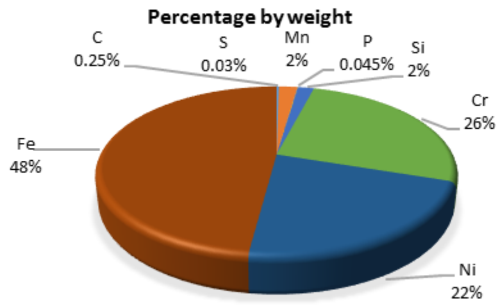

Figure 5. Chemical composition of AISI 310 stainless steel.

Alloy 309/309S and alloy 310/310S austenitic stainless steel are typically used for high temperature applications. Its high chromium and nickel content provides comparable corrosion resistance, superior resistance to oxidation and retention of a higher fraction of resistance at room temperature than common austenitic alloy 304.
Metallurgical instability or the formation of new phases during exposures to high temperatures can adversely affect the mechanical properties and reduce the resistance to corrosion. Carbide particles tend to precipitate at grain boundaries (sensitization) when austenitic stainless steels are held in or cooled slowly over a temperature range of 800-1650°F (427-899°C). The higher levels of chromium and nickel contained in these alloys result in lower carbon solubility, which tends to increase susceptibility to sensitization. It is recommended to cool with an immersion choke (gas or liquid) for this critical temperature range, particularly for thicker sections. The time at the temperature required to form chromium carbides increases as the carbon content is reduced. Therefore, the low carbon versions of these alloys are more resistant but not immune to sensitization. When heated to temperatures of 1200-1850 F (649-1010°C) for extended periods, alloys 309/309S and 320/320S may exhibit reduced ductility at room temperature due to the precipitation of brittle particles from the second phase (sigma phase and carbides). The sigma phase is often formed at the grain boundaries and can reduce ductility. This effect is reversible and the complete ductility can be restored by re-annealing at the suggested temperatures.

3. Results and discussions

The first data that must be obtained for the theoretical predictions, are based on the braking forces in the motorcycle taking into account the mass (weight in running order), and the approximate mass of the pilot, as shown in Table 2:

Table 2. Data for the calculation of the braking force of the three motorcycles.

Calculation	Suzuki Best.	AKT CPI	Suzuki GS
Mm (kg)	96.7	98	134
Pm (kg)	70	70	70
g (m/s^2)	9.81	9.81	9.81
Tm (Kg)	166.7	168	204

Then, it is necessary to calculate the distance from the center of gravity of the motorcycle to the front axle, which is where the disk is located. For this it is necessary to sum up the moments in the front wheel and calculate the weight of the rear wheel which is approximately 10 kg for the Suzuki Best and AKT CPI, and 12 kg for the Suzuki GS. The sum of moments is:

$$\Sigma \mathrm{M\ front\ wheel} = (\mathrm{P\ rear\ wheel} \times \mathrm{\ d\ between\ axes}) - (\mathrm{P\ total\ motorcycle} \times \mathrm{\ D\ gravity\ center}) \quad (\mathrm{Eq.}\,1)$$

The distance from the center of gravity to the front axle is 496 mm, as shown in Figure 6.

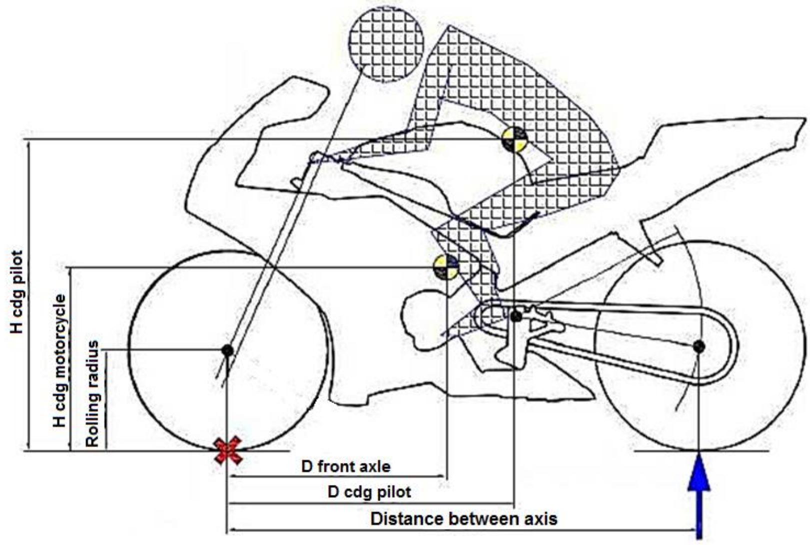

Figure 6. Center of gravity of Suzuki Best motorcycle (Zurin, Talib and Ismail, 2017).

For the analysis of braking forces in motorcycles, is necessary to use the distance values of the centers of gravity, are shown in the following Table 3:

Table 3. Distance values of centers of gravity for the three motorcycles.

DISTANCES OF GRAVITY CENTERS (front wheel reference point) (mm)			
Calculation	**Suzuki Best.**	**AKT CPI**	**Suzuki GS**
$h_{cdg\ p}$	740	765	773
$d_{cdg\ p}$	791	700	869
$d_{front\ axle}$	496	439	545
$h_{cdg\ m}$	550	569	574
d_{axis}	1240	1225	1340
μ	0.9	0.9	0.9
Pill coefficient	0.45	0.45	0.45
Rolling radius (mm)	280	280	298
Diameter of the disc (mm)	220	220	275

The braking forces have many variables, such as the speed of the motorcycle, the acceleration, the coefficient of friction of the pavement and the pad, the mass of the motorcycle and the pilot, the braking distance, among other factors. Table 5 shows the values of the braking force, it is calculated whether or not there is blockage on the wheel when braking, acceleration, braking time and braking deceleration on several slopes of a track, using constant data such as speed (80 km/h) and braking distance.

Acceleration: It is calculated taking into account the following Equation 2:

$$a = \frac{V_i^2 - V_f^2}{2 \times D} \quad (\text{Eq. } 2)$$

The braking time: To carry out this calculation, the following Equation 3:

$$t = \frac{2 \times D}{V_i^2 + V_f^2} \quad (\text{Eq. } 3)$$

Braking force: This calculation is given by the following Equation 4:

$$F = \left(M_m + M_p\right) \times a \quad (Eq. 4)$$

Also, to check if there is a blockage on the front wheel when braking at a certain speed and braking distance, the following relationship of Equation 5 should be made:

$$\left(M_m + M_p\right) \times a < P_{front\ wheel} \times \mu \quad (Eq. 5)$$

In the event that the coefficient of friction between the tire and the ground is less than required, the front wheel will lock and begin to slide on the pavement. To find the weight on the front wheel it is necessary to sum up moments at the point where the front wheel makes contact with the ground and find the reaction with the ground (Rb) on the rear wheel. This is as follows:

$$\sum MA = 0$$
$$(-M_m \times g \times d_{cgd\ m}) - (M_p \times g \times d_{cgd\ p}) + (M_m \times a \times h_{cdg\ m}) + (M_p \times a \times h_{cdg\ p}) + (R_b \times d_{axis}) = 0$$

Then, when Rb is cleared, the weight must be calculated on the front wheel, and this is calculated as follows:

$$P_{front\ wheel} = (M_m \times g) - R_b$$

It was taken into account the weight on the front wheel, to check if there is a blockage or not. It is was determined on the motorcycle Suzuki Best 125cc, the lock on the front wheel is presented at a braking distance of 185 m. The AKT CPI 110cc motorcycle has a lock on the front wheel at a braking distance of 105 m, the Suzuki GS 150cc motorcycle has a lock on the front wheel at a braking distance of 95 m, considering that's motorcycles is going at a speed of 80 km/h and that the rear brake is not performing its function. The Tables 5, 6 and 7 show the calculations made for each of the disc brakes. On the other hand, it can be seen that the greater the cylinder capacity of the motorcycles, the value of each of the calculations made, related to the geometry of each of the disks, increases.

Table 5. Calculation of braking distance Suzuki Best.

D (m)	a (m/s)	t (s)	F (N)	Rb (N)	Wf (N)	Checking the wheel lock		Exists blocking	Tf (N)	Braking deceleration (m/s^2) (grades (θ))			
						$(M_m + M_p) \times a$	$W_F \times \mu$			0	5	10	20
210	1.18	18.90	-196.00	717.95	230.68	196.00	207.61	No	1174.35	10.00	10.86	11.71	13.36
205	1.20	18.45	-200.78	715.52	233.10	200.78	209.79	No	1186.71	10.03	10.89	11.74	13.39
200	1.23	18.00	-205.80	712.97	235.65	205.80	212.09	No	1199.69	10.06	10.92	11.77	13.42
195	1.27	17.55	-211.08	710.29	238.33	211.08	214.50	No	1213.33	10.10	10.95	11.80	13.45
190	1.30	17.10	-216.63	707.47	241.15	216.63	217.04	No	1227.70	10.13	10.98	11.83	13.48
185	1.33	16.65	-222.49	704.50	244.13	222.49	219.72	Si	1242.84	10.16	11.02	11.87	13.52
180	1.37	16.20	-228.67	701.36	247.27	228.67	222.54	Si	1258.81	10.20	11.06	11.90	13.56
175	1.41	15.75	-235.20	698.04	250.59	235.20	225.53	Si	1275.71	10.24	11.09	11.94	13.60

Table 6. Calculation of braking distance AKT CPI

D (m)	a (m/s)	t (s)	F (N)	Rb (N)	Wf (N)	Checking the wheel lock		Exists blocking	Tf (N)	Braking deceleration (m/s^2) (grades (θ))			
						$(M_m + M_p) \times a$	$W_F \times \mu$			0	5	10	20
130	1.90	11.70	-319.09	567.44	393.94	319.09	354.55	No	2005.5	10.72	11.58	12.43	14.08
125	1.98	11.25	-331.85	560.66	400.72	331.85	360.65	No	2040.0	10.80	11.65	12.50	14.15
120	2.06	10.80	-345.68	553.32	408.06	345.68	367.26	No	2077.4	10.88	11.74	12.59	14.24
115	2.15	10.35	-360.71	545.33	416.05	360.71	374.44	No	2118.0	10.97	11.83	12.67	14.33
110	2.24	9.90	-377.10	536.63	424.75	377.10	382.28	No	2162.3	11.07	11.92	12.77	14.42
105	2.35	9.45	-395.06	527.09	434.29	395.06	390.86	Si	2210.9	11.18	12.03	12.88	14.53
100	2.47	9.00	-414.82	516.60	444.78	414.82	400.31	Si	2264.3	11.29	12.15	13.00	14.65
95	2.60	8.55	-436.65	505.00	456.38	436.65	410.74	Si	2323.3	11.42	12.28	13.13	14.78

Table 7. Calculation of braking distance Suzuki GS.

D (m)	a (m/s)	t (s)	F (N)	Rb (N)	Wf (N)	Checking the wheel lock		Exists blocking	Tf (N)	Braking deceleration (m/s^2) (grades (θ))			
						$(M_m + M_p) \times a$	$W_F \times \mu$			0	5	10	20
120	2.06	10.80	-419.75	778.78	535.76	419.75	482.18	No	2322.2	10.88	11.74	12.59	14.24
115	2.15	10.35	-438.00	770.03	544.51	438.00	490.06	No	2360.1	10.97	11.83	12.67	14.33
110	2.24	9.90	-457.91	760.49	554.05	457.91	498.65	No	2401.5	11.07	11.92	12.77	14.42
105	2.35	9.45	-479.72	750.04	564.50	479.72	508.05	No	2446.8	11.18	12.03	12.88	14.53
100	2.47	9.00	-503.70	738.54	576.00	503.70	518.40	No	2496.6	11.29	12.15	13.00	14.65
95	2.60	8.55	-530.22	725.83	588.71	530.22	529.84	Si	2551.7	11.42	12.28	13.13	14.78
90	2.74	8.10	-559.67	711.72	602.82	559.67	542.54	Si	2612.9	11.57	12.42	13.27	14.92
85	2.90	7.65	-592.59	695.94	618.60	592.59	556.74	Si	2681.3	11.73	12.58	13.43	15.08

In addition, the conditions for the braking distance calculations were a initial speed of 80 km/h and final speed of 0 km/h. The following Figure 7, shows the braking distances for each of the three-disc brakes:

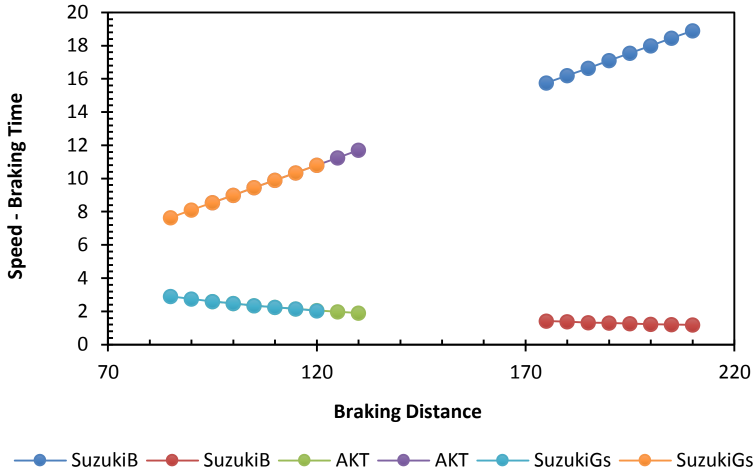

Figure 7. Calculation of the braking distance for the three motorcycles.

The braking distances for the three discs vary in a matter of thousandths of a second, so and the three distances between the three motorcycles must be averaged for the calculation question. For the case of the three discs, the braking time varies according to the speed interval influenced by the action of braking time and distance. In addition, it demonstrates the effectiveness of the system in its different operating conditions.

These braking times are found in 7.6 and 11.7 s for a braking speed of 1.9 and 2.9 m/s. The opposite is the case with the disc brake of the smallest motorcycle, where the braking times and speeds are between 15.7 and 18.9 s for a speed of 1.1 and 1.4 m/s.

Furthermore, the reaction time of the system is not considered t_{RS}, is the time that elapses since the brake pedal is actuated until the required force is reached, this is an approximate value of 3 seconds. On the other hand, aerodynamic actions are not taken into account, as aerodynamic drag is effective in the vehicle from more than 90 km/h. For the braking distances according to the inclination angles, the following results analysis is presented:

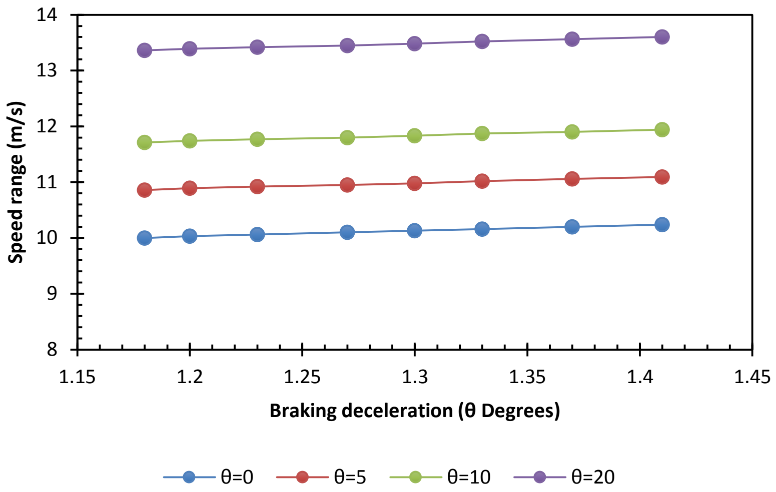

Figure 8. Braking distances at varying speeds up to stop and angles of descent for the motorcycle Suzuki Best.

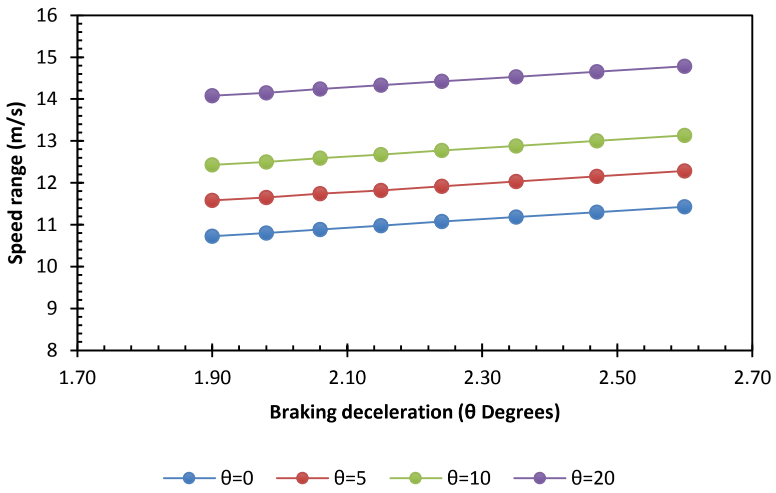

Figure 9. Braking distances at varying speeds up to stop and angles of descent for the motorcycle AKT.

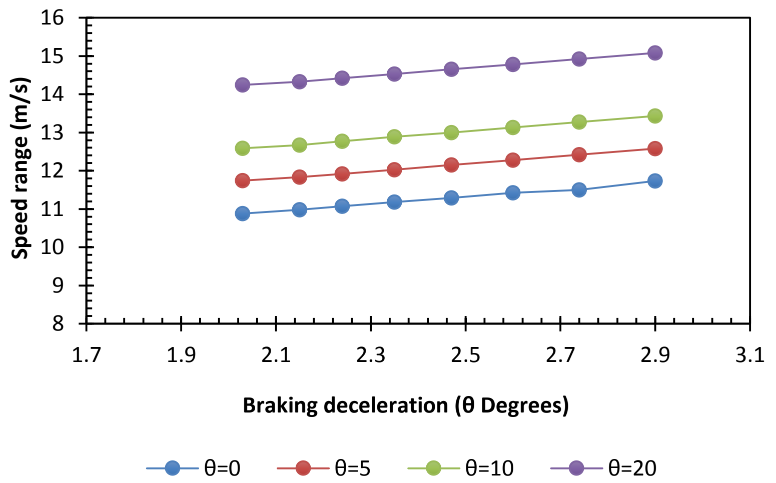

Figure 10. Braking distances at varying speeds up to stop and angles of descent for the motorcycle Suzuki Gs.

It can be seen in Figures 8, 9 and 10, that the braking distance is influenced by the motorcycle cylinder, as can be seen in the braking efficiency of the highest cylinder motorcycles of 135 and 150 cc, which oscillates between 85 and 130 m for both cases also related as a simultaneous effect between time and braking speed also decreasing every time a measurement is also checked the angle of inclination or descent of the road; As well as the gravitational drag that drives the vehicle, the braking distance is also seen with aerodynamic characteristics such as the frontal area and the coefficient of drag, factors can include factor C is included in the logarithmic function for each case, thus the larger the sea the front area of the vehicle will be the coefficient C and thus have less braking distance.

The results presented on the three disc brake, are similar to those reported in some bibliographic references because there is little information reported on the disc brakes of motorcycles. On the other hand, the dynamic behavior of motorcycles is similar to that of an automobile, but the differences are the type of tire, power, and load capacity, among others.

4. Conclusions

The system that the authors of the research considered a reason for study and calculation was the most appropriate, because the elements under consideration are easily accessible and nowadays almost all motorcycles have disc brakes ventilated in the two wheels. Otherwise, it occurs in motorcycles that continue to use the drum brakes on the rear axle, and usually most of the drawbacks such as the loss of heat, moisture and thus the corrosion that result from the braking inefficiency, because the areas of heat generation are closed and the heat evacuation is delayed.
This research is conducted to study the relationship of the geometry of brake disc towards the better braking performance in term of heat dissipation to surrounding. From the results, it shows that the greater the cylinder capacity the braking effectiveness will be, so the disc brake 3 is considered to have the best dynamic properties for the system.
Finally, it is of great importance the application of design Software for Finite Element Analysis (FEA), because it is possible to check, verify and analyze working conditions of the disc brakes, as well as to apply dynamic theory, of which it was evident that these braking systems start from the pedal force.

5. References

Carro Suárez, J. *et al.* (2017) 'Modelo de desarrollo sustentable para la industria de recubrimientos cerámicos', *Revista Internacional de Contaminacion Ambiental*, 33(1), pp. 131–139. doi: 10.20937/RICA.2017.33.01.12.

Cengel, Y. (2007) *Tansferencia de calor y masa. Un enfoque práctico. Tercera edición.* México: McGraw-Hil.

Conraud Koellner, E. and Arredondo Hidalgo, M. G. (2018) *Desempeño Sustentable De La Industria Automotriz En México: Nuevas Perspectivas De Creación De Valor Para El Consumidor Millenial*. Mexico.

Criado, E. (2012) *Diseño y cálculo del sistema de frenado para un prototipo formula student*. Universidad Carlos III de Madrid.

Dhaubhadel, M. N. (1996) 'Review: CFD Applications in the Automotive Industry', *Journal of Fluids Engineering*, 118(4), p. 647. doi: 10.1115/1.2835492.

García-León, R. A. (2014) *Evaluación del comportamiento de los frenos de disco de los vehículos a partir del análisis de la aceleración del proceso de corrosión.* Universidad Francisco de Paula Santander Ocaña.

García-León, R. A. (2017) 'Thermal study in three vented brake discs, using the finite element analysis', *DYNA (Colombia)*, 84(200), pp. 19–27. doi: http://dx.doi.org/10.15446/dyna.v84n200.55663.

García-León, R. A. and Flórez-Solano, E. (2017) 'Dynamic analysis of three autoventilated disc brakes', *Ingeniería e Investigación*, 37(3), pp. 102–114. doi: 10.15446/ing.investig.v37n3.63381.

García-León, R. A., Flórez-Solano, E. and Acevedo-Peñaloza, C. (2018) *Análisis termodinámico en frenos de disco*. Bogota, Colombia: ECOE Ediciones.

García-León, R. A., Flórez-Solano, E. and Rodriguez-Castilla, M. (2019) 'Thermo-mechanical assessment in three auto-ventilated disc brake by implementing finite elements', *J. Phys. Conf. Ser.*, 11, pp. 11–29.

García-León, R. A. and Perez Rojas, E. (2017) ‘Analysis of the amount of heat flow between cooling channels in three vented brake discs’, *Ingenieria y Universidad*, 21(1), pp. 55–70. doi: 10.11144/Javeriana.iyu21-1.aahf.

Manohar Reddy, S., Mallikarjuna, J. M. and Ganesan, V. (2006) ‘Flow and heat transfer analysis through a brake disc - A CFD approach’, *American Society of Mechanical Engineers, Heat Transfer Division, (Publication) HTD*, pp. 1–5. doi: 10.1115/IMECE2006-14317.

Pan, L. *et al.* (2015) ‘Numerical simulation for train brake disc ventilation’, *Beijing Jiaotong Daxue Xuebao/Journal of Beijing Jiaotong University*. School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing, China, 39(1), pp. 118–124. doi: 10.11860/j.issn.1673-0291-2015.01.020.

Talati, F. and Jalalifar, S. (2009) ‘Analysis of heat conduction in a disk brake system’, *Heat and Mass Transfer*, 45(8), pp. 1047–1059. doi: 10.1007/s00231-009-0476-y.

Todeschini, F. *et al.* (2014) ‘Adaptive position-pressure control of a brake by wire actuator for sport motorcycles’, *European Journal of Control*. Elsevier, 20(2), pp. 79–86. doi: 10.1016/j.ejcon.2013.12.003.

Zurin, W. M. W. S., Talib, R. J. and Ismail, N. I. (2017) ‘Thermal analysis on motorcycle disc brake geometry’, *AIP Conference Proceedings*, 1875. doi: 10.1063/1.4998393.